POWER IN MOTION

Horse Power, Wheel Gearing, Driving Bands, and Angular Forces.

By JAMES ARMOUR, C.E.

With numerous Illustrations

LONDON:
LOCKWOOD & CO., 7, STATIONERS' HALL COURT,
LUDGATE HILL.
1871.

LONDON:
PRINTED BY VIRTUE AND CO.,
CITY ROAD.

PREFACE.

In this short treatise the general laws that govern power in motion are exhibited in simple form, to meet the known wants of practical men engaged in engineering works that require the employment of horses, hoists, block and tackle, wheel gearing, and long and short driving bands, of wire rope or of leather.

The aim is, so to explain the essential principles, that the value of means proposed for the performance of work, under the varied circumstances of common practice, may be easily determined.

Common arithmetic alone is used in working out the questions, except in the case of driving bands, when, for the extraction of certain values, common logarithms are employed necessarily, but in so simple a manner that no difficulty need be felt, even by those unaccustomed to the use of logarithms.

In treating questions of angular force a table of natural sines, &c., is necessary, where the value is decided by the angle; but the simple method of computing by the tabular values of the angle has been so circumstantially explained, and exhibited so plainly in the illustrations, that no difficulty need occur here either.

GATESHEAD,
April, 1871.

POWER IN MOTION.

SECTION I.

1. Power, and work done, stand relatively to each other as cause and effect.

Pressure is power in an active state.

Resistance, without which pressure cannot exist, is power in an inert state.

When the resistance equals the pressure, there is no work done, because of the balance between the forces. When the resistance is less than the pressure, the resisting body is pressed out of its place, and the extent of its motion multiplied by its resistance, gives the measure of the work done by the moving power.

2. As it has been found convenient to adopt a unit of length, termed 1 foot, for measuring distance, and a unit of weight, termed 1 lb., for measuring gravity, and a unit of time, termed 1 minute, for measuring time; so it has been found necessary in measuring forces, to adopt what is termed a unit of work, which is performed whenever a pressure of 1 lb. is exerted through the space of 1 foot in any direction; that is, either in lifting or in pushing.

We must distinguish, however, between mere deadweight suspended, and the same when rolling or sliding: thus, the pressure exerted through the space of 1 foot will, in the case of sheer lifting, be simply not less than the weight lifted; whereas in a case of pushing or pulling,

the pressure or tension will simply be not less than the frictional resistance of the load in motion; and this, as we shall find shortly, may amount in some instances to merely a very small fraction of the total load.

3. When we raise vertically a pound weight 1 foot high, we perform 1 unit of work, and when we raise it 10 feet, we perform 10 units of work. Whereas, when we slide a pound weight along a surface, the frictional resistance between which and the weight is equal to, say, one-fifth or ·20 of the whole weight, we do not perform 1 unit of work until we have slidden the weight 5 feet, and have to slide it 50 feet before we have performed 10 units of work.

4. The time occupied in these several motions does not require to be known in determining simply the number of units of work performed; so that the lift of 10 feet may take 10 seconds or 10 hours, without affecting the measure of the power expended; but, when we wish to know the capacity for work of any source of motive power, we must ascertain how many units it is capable of doing in a given time, say in 1 minute.

We can easily see that without a time-standard in questions of motion, we would be unable to form any clearer idea of the value of any given motive power, than we might get by merely weighing in a pair of scales, the pressure exerted, and the resistance overcome. We would know the slight difference between them, but not what it was worth practically.

A time standard, however, to be of practical use, must refer to the capacity for work, or the strength, of some definite power with which we are familiar, such as the strength of an average man, when manual labour is concerned, or is about to be superseded by engine power; or of a horse when the forces are of greater magnitude, or such as usually, or previously, were supplied by horses.

5. It has been shown by experiment that a horse or

average strength will perform about 22,000 units of work per minute.

When an average horse is actually employed to do the work, the standard ought probably to be no higher than this; but when the power is derived from steam or water, we may use the number 33,000 given by Watt, and still used by engineers; because, though this latter number be in excess, it serves quite as well for a standard as the natural strength of the horse, when it refers simply to pressure putting weight in motion.

6. Watt, in his estimate, assumed the ordinary pace of the horse to be at the rate of $2\frac{1}{2}$ miles an hour, which is equal to $3\frac{2}{3}$ or 3·66 feet per second, or 220 feet per minute; and that at this pace a heavy London dray-horse could raise 150 lbs. suspended at *b* by a rope passing over a pulley *a*, Fig. 1.

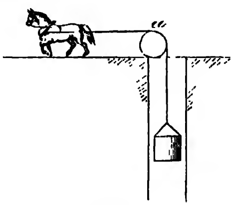


Fig. 1.

$$\frac{\overset{\text{Miles.}}{2\frac{1}{2}} \times \overset{\text{Feet.}}{5280}}{60 \text{ minutes}} = 220 \text{ feet raised per minute, the height of}$$

the lift being equal to the distance walked by the horse: so that 150 lbs. × 220 feet per minute = 33,000 lbs. raised 1 foot high, or 1 lb. raised 33,000 feet high per minute, the force expended being the same whether we make the unit figure 1 lb. or 1 foot.

7. Now, as a pressure of 1 lb. exerted through a space of 1 foot in any direction, is the standard measure for a unit of work, so these 33,000 units form the recognised unit of horse-power, expressed thus: 33,000 units of work = 1 H.P., referable, however, to 1 minute of time, because the unit of horse-power is the unit of intensity; whereas the unit of work is simply the unit of quantity, without reference to time. So that the latter may be performed by a very weak power exerting a pressure of,

say, one-twelfth of a pound through a distance of 12 feet, or by a relatively strong power exerting a pressure equal to 12 lbs. through a distance of 1 inch. Either of these quantities of work performed being simply equal to 1 lb. pressure exerted through a distance of 1 foot, and therefore equal to 1 unit. Thus:—

	Feet.		lbs.		Units.
(1)	12	$\times$	$\frac{1}{12}$	=	1
(2)	$\frac{1}{12}$	$\times$	12	=	1
(3)	1	$\times$	1	=	1

When time is taken into account the (2) case would require to move 12 times as slow and the (1) case 12 times as quick as case (3) in performing 1 unit of work in the same time, but as this introduces velocity into the question as a new element, which will vary with every slight change in the pressure, we save trouble by limiting the time to 1 minute for the performance of 33,000 units; so that when we have the pressure in pounds, which can never be sensibly more than the resistance when the motion is uniform, we divide the standard 33,000 by it and thereby get the distance in feet travelled by the pressure within the minute of time, and consequently, the velocity in feet per minute.

8. We have already found that when the resistance equals 150 lbs., the velocity is 220 feet per minute.

When the resistance is increased to 200 lbs. we have $33{,}000 \div 200 = 165$ feet per minute, and similarly for other pressures; the speed decreasing as the resistance increases, and increasing as the resistance diminishes: because, the unit force of a nominal horse-power being a definite quantity, which is assumed to be fully employed when lifting 150 lbs. at the rate of 220 feet per minute, that definite quantity would be only half expended were we with the same load to reduce the speed to 110 feet per minute, and it would have twice the work it was

capable of performing were the speed doubled by increasing it to 440 feet per minute.

We may see this in clearer light when we use simpler terms, thus:—A unit of work represents 1 lb. raised 1 foot in, say 1 minute; if that 1 lb. be raised only ½ foot in 1 minute, there is only ½ unit of work performed. Consequently the power that is capable of doing a whole unit in that time is exerting only half its strength.

Again, if the 1 lb. weight has to be raised 2 feet in the minute, equal to 2 units of work, by the power that is capable of doing only 1 unit, it is plain that either the power must be doubled or the weight must be reduced to ½ lb., to bring the work down to the 1 unit capacity of the power.

When the speed is reduced to 110 feet per minute, we have 110 feet × 300 lbs. = 33,000 units, and when it is increased to 440 feet per minute, we have 440 feet × 75 lbs. = 33,000 units.

9. When we know the distance only, we divide the 33,000 by it to get the pressure; and, as before remarked, when we know the pressure only, we divide the 33,000 by it to find the distance; that is, when a clear and simple horse-power is concerned.

When we do not know the value of the power, that is, the number of units of work it can expend per minute, we ascertain it by multiplying the distance by the resistance overcome, and then dividing the product by 33,000; in which case the velocity and resistance have first to be determined.

10. With reference to our remark that the force expended in performing 1 H.P. of work is the same whether we say 33,000 lbs. 1 foot high, or 1 lb. 33,000 feet high, we may make the reason sufficiently plain by stating the case thus: 1 lb. raised 1 foot high being 1 unit, it is clear that there will be as many units as there are feet in any height to which that pound weight may be carried;

and again, when we say 33,000 lbs. raised 1 foot = 33,000 units, we have only to suppose each pound weight lifted separately to the height of 1 foot. So that 33,000 feet × 1 lb. = 33,000 units, and 1 foot × 33,000 lbs. = 33,000 units.

This direct method, however, must be understood to apply to nominal horse-power only, as exerted by inanimate agency, such as steam or water-power. When speed is increased in the case of a horse yoked, as in Fig. 1, the power of the animal is expended not only in raising the pulley load at a quicker rate, but also in carrying the weight of its own body quicker over the ground; hence it follows that, when speed is increased, the pulley load must be lightened in a different proportion to what is required when the power is derived from steam, which is pure pressure or power without appreciable weight of its own requiring to be carried; or, when the power is derived from water, the pressure from which is due solely to the weight of the fluid, which being borne solely by the resistance, experiences nothing corresponding with the fatigue of the horse, but, gravitating ever downwards, performs work by its motion, the more motion the more work, instead of absorbing power to supply its own wants as in the case of the weight of the horse.

11. To put the power of the horse on equal terms with the water, we would have to put the animal into a cage, as in Fig. 2, and let the simple weight of its body act in place of the lean-to pressure of that weight aided by the muscular power, as in Fig. 1.

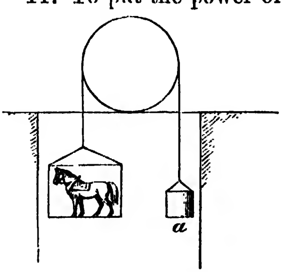


Fig. 2.

Placed in the cage, it would raise any load *a* not quite equal to its own weight, and would thus raise a greater weight than when pulling on

the ground. But then, like the water, it would be useless for further effort when it reached the bottom that it had raised the load *a* from.

12. Further, to put it on equal terms with steam-power, we would require the body to communicate all the energy of its muscular power to bodies outside of itself, without abstracting any to enable its own motions to be made. The energy of the muscular power, however, does not comprise the whole power of the horse, because, much of the whole power is due to the weight of the body; so that, if we were to make that weight no more than equal to the 150 lbs. load, lifted at the rate of 220 feet per minute, the muscular power, no matter what its amount might be, could work with no more effect than afforded by the friction of 150 lbs. dead weight pressure upon the ground, so that if the friction of the horse's feet upon an ordinarily smooth road amounted to, say, one-fourth of the weight of body they supported, we would require to saddle weights upon his back, to make the pressure of the feet equal to four times the pulley load, to just save him from sliding backward; because the friction of his assumed weight, 150 lbs., being only $150 \div 4 = 37{\cdot}5$ lbs., the pulley load has four times his power, and would quickly overcome him. (Paragraph 19.)

The working parts of a steam-engine, or of a water-wheel, require to be put in motion by the motive power, equally with the body of the horse at work, and friction is increased in these working parts when the burden of work to be done is increased, equally with what happens in the joints of a horse when a greater burden is put upon him: but, the horse's joints are softer than the shaft-bearings, wheel-teeth, and crank-pins, of an engine, and they are steadied by muscles which, though admirably disposed and strung together, are more easily injured than things of wood and iron, so that one horse-power of animal strength acts within a more limited range than

one horse power nominal in the case of steam or water-power.

13. Tredgold adopts the 33,000 unit standard, but estimates that the horse performs this maximum amount of work when moving at the rate of 3 miles an hour, consequently, the load he then lifts is less than the 150 lbs. for the 2½ mile rate according to Watt. He makes allowance for decrease in the strength of the animal at higher speeds, and for a corresponding increase at lower speeds than his 3 mile standard rate.

We will put a few of his figures into form here.

Miles per hour.		Feet per min.		Pounds lifted.		Units per min.
2	=	176	×	166	=	29216
2½	=	220	×	146	=	32120
3	=	264	×	125	=	33000
3½	=	308	×	104	=	32032
4	=	352	×	83	=	29216
5	=	440	×	41⅔	=	18330

14. The Rule by which the load is ascertained when the speed is known is thus expressed.

250 — (41⅔ × the speed in miles) = the load lifted by pulling in pounds. Thus :—

$$250 - 41{\cdot}66 \times 3 = 125 \text{ lbs.}$$

The 41·66 is multiplied by the number of miles per hour before being subtracted from the 250.

We shall have occasion, later on, to speak more fully regarding the full load, which is here represented by 125 lbs. pulling or lifting force, but may here observe that this Rule is only approximately correct. The constants 250 and 41·66 are applicable only to speeds not greater than 5 miles an hour. Moreover, the results they give are indefinite as to the length of time the animal is capable of continuing the exertion.

This is of little consequence, when we seek merely a standard for steam or water-power, without particular

reference to the animal power to be superseded. But as our remarks refer more particularly to animal power, we will now employ a Table of Tredgold's relating to the day's work of a horse of average strength, performed at different velocities, on common roads and railways, and in towing boats on canals.

15. Useful effect of one horse working one day, in tons drawn one mile.

Velocity per hour.	Duration of the day's work.	Pulling Force.	Canal.	Level Railway.	Level Road.
Miles.	Hours.	lbs.	Tons.	Tons.	Tons.
$2\frac{1}{2}$	$11\frac{1}{2}$	$83\frac{1}{3}$	520	115	14
3	8	,,	243	92	12
$3\frac{1}{2}$	$5\frac{9}{10}$	,,	153	82	10
4	$4\frac{1}{2}$	,,	102	72	9
5	$2\frac{9}{10}$	,,	52	57	7·2
6	2	,,	30	48	6·0
7	$1\frac{1}{2}$	,,	19	41	5·1
8	$1\frac{1}{8}$	,,	12·8	36	4·5
9	$0\frac{9}{10}$	,,	9·0	32	4·0
10	$0\frac{3}{4}$	,,	6 6	28·8	3·6

16. We here perceive that when the speed is increased, the load in tons decreases, in the case of canal towage, at a quicker rate than in the case of railway and common road draught. This is owing to the boat in motion having to displace, for every mile it advances, a body of water equal to the average breadth and depth of the body of the boat under water, multiplied by 1 mile length; whereas rolling loads have merely the surface resistance of the roads and the friction of the wheel axles to overcome.

We will suppose, for convenience in calculation, that the weight of that volume of water, 1 mile in length, is only 520 tons, equal to the weight of the boat for the one-horse power at the $2\frac{1}{2}$ mile speed.

Of course, the weight of the body of water 1 mile long would be immensely greater than this, but we will assume it to be no more than the weight of the boat, because we

know that a boat afloat displaces a weight of water equal to its own weight, and its own weight being here known, and being at the same time the measure of the draught upon the horse-power, we use it in finding simply how the horse-power stands in relation to it at different speeds, as regards resistance only. When we wish to ascertain the total actual work done, in towing the boat 1 mile, we require to find the resistance of the whole water displaced in that distance.

17. When we double the speed from the rate of 2½ miles an hour to 5 miles, we displace double the volume of water in the same time as was occupied in the displacement of the single mile volume at the lower speed; that is, we have to displace twice the weight of water, at twice the velocity, which gives us $2 \times 2 = 4$ times the resistance, which is equivalent to 2080 tons of water displaced at 2½ miles an hour; so that 4 horse-power would be required at 5 miles an hour to tow the load that required only 1 horse-power at 2½ miles; and this increase of power from 1 to 4 would be sufficient to maintain uniformly, for any length of time, the 5 mile speed of a boat weighing 520 tons, were it derived from a steam-engine on board the boat: but, when the power works in the body of the horse, which has to apply an increased proportion of its strength to carry its own weight forward at the increased pace, we find that the strength of the 4 horses would be spent quickly, and long before they had expended upon the boat the full unit power of work they possessed capacity for at the 2½ mile pace. (Paragraph 35.)

We find that the Table gives as a day's work of 2·9 hours for 1 horse, only 52 tons, drawn 1 mile at the rate of 5 miles an hour. This is only one-tenth of the load which a horse can draw 1 mile at 2½ miles an hour for a day's work of 11½ hours.

As the Tables relate to the work of 1 horse only, and

as the duration of a day's work at the $2\frac{1}{2}$ mile speed is 11·5 hours, we have $11{\cdot}5 \times 2{\cdot}5 = 28{\cdot}75$ miles distance travelled; and, at the five-mile speed, 2·9 hours $\times\ 5 = 14{\cdot}5$ miles; so that $28{\cdot}75 \div 14{\cdot}5 = 2$ times nearly, distance ratio. And, as 520 tons $\div\ 28{\cdot}75 = 18{\cdot}09$ tons nearly, per mile for the $2\frac{1}{2}$ mile speed; and 52 tons $\div\ 14{\cdot}5 = 3{\cdot}58$ tons per mile for the 5-mile speed: we have $18{\cdot}09 \div 3{\cdot}58 = 5$ times, load ratio: so that, $5 \times 2 = 10$ times, the same as got directly thus, $520 \div 52 = 10$.

A load of 18·09 tons hauled 28·75 miles is equivalent to the convenient tabular expression, 520 tons hauled 1 mile; and 3·58 tons hauled 14·5 miles is equivalent to 52 tons hauled 1 mile.

In this we assume that the powers given in the Table are correct: not absolutely, though sufficiently near for approximate estimate.

18. In the case of the highest speed of 10 miles an hour, 3·6 tons drawn 1 mile on a level road in three-quarters of an hour, is the estimated day's work. Now, this statement of speed, load, and time, thus expressed in tabular form, has its equivalent in about half a ton drawn $7\frac{1}{2}$ miles in the same time. The principle on which the Table is framed, allows this equivalent to hold good, but we have to suppose the load and the horse-power divided each into $7\frac{1}{2}$ parts, each of these fractional powers bearing forward in the time given, a corresponding fraction of the load the distance of 1 mile, to make the aggregate equal to $7\frac{1}{2}$ miles with $\frac{1}{7\frac{1}{2}}$ portion of the load; in the same way as we imagined the pound weights of the 33,000 lbs. to be all separately raised 1 foot high, to form in the total 1 horse-power.

Seven and a half miles in three-quarters of an hour is at the rate of 10 miles an hour, the usual speed of mail stage-coaches on a level road.

These coaches, when loaded, weigh about 2 tons,

consequently, the 4 horses usually yoked to them are no more than necessary to draw them at the 10 mile rate; and those horses have done work enough for one day, when they have performed a $7\frac{1}{2}$ mile stage with a half-ton load each, at that pace. (Paragraph 35.)

SECTION II.

19. A horse, when in the act of pulling, leans its weight forward until the centre of gravity of the weight so far overhangs the feet as to overcome the resistance of the load behind it.

Let the block *x x*, Fig. 3, represent the body of the horse, resting on legs *t t*. The weight of the body aided

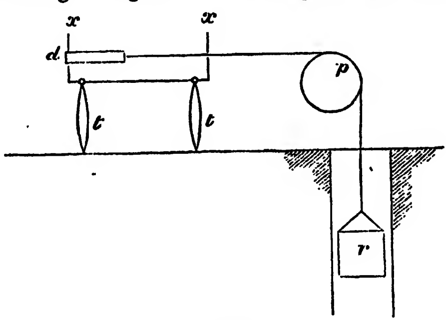


Fig. 3.

by the muscular strength has to raise the load *r*, by means of a rope passing over the pulley *p*.

Now, it is clear that when the horse is standing at ease, the weight of the body is simply supported, that is, it is merely acting with a downward pressure, in which condition the pulley load would have little difficulty in drawing it out of balance backward, and if, as we before observed, the load be greater than the amount of friction of the horse's feet upon the ground, would quickly slide it backward bodily, supposing the legs rigid.

20. We will assume the weight of the horse to be only 900 lbs., and the pulley load 150 lbs. clear strain.

The pressure of power required to overcome the resistance of the pulley load is derived from the weight of the animal, the muscular strength of the limbs being required for motion mainly.

The heavier the strain upon the horse, the lighter is the burden upon the fore limbs, because in pressing forward upon the breast-band *d*, that band, connected to the rope that bears the load, receives, and supports, as much of the weight as is required to overcome the resistance, leaving the fore limbs no more to support than a part of the weight of the fore half of the body. The hind limbs, on the other hand, being more distant from the point *d*, have to support nearly their ordinary share of one-half of the full weight of the body; the hind feet are thereby planted firmer on the ground, and, as the hind limbs, by formation and situation, are best adapted for propelling the body forward against the pressure at *d*, this greater weight upon them is an advantage.

21. The pressure required is 150 lbs.; we find the

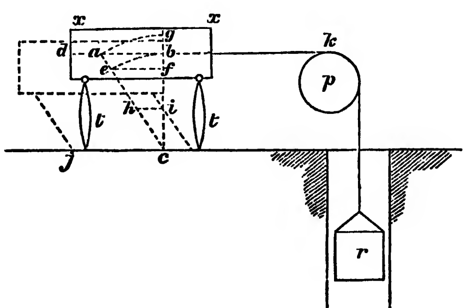


Fig. 4.

number of times this is contained in the 900 lbs. weight of body; then divide the height *d j*, Fig. 4, by this

number, to get the distance *a b*, which the weight of the body must project in advance of the feet, to give the pressure that will overcome the load resistance. For convenience, we place the triangle of which *a b* is to form a side, in the hinder half of the block.

900 ÷ 150 = 6 times, to be used as a divisor of the height of the horse, which we will assume to be 54 inches, represented by the line *b c*, which is made equal to *d j*, the whole line *d a b k* being horizontal and parallel with the ground.

54 ÷ 6 = 9 inches = *ab*, which represents 150 lbs. resistance, when *b c* represents 900 lbs.

In the Fig., the angle is exaggerated to make the parts more distinct.

22. In projecting the centre of gravity of the weight from *b* to *a*, with *c*, which represents the feet of the horse, kept stationary as a centre, the height *b c* will be reduced to an extent which may be measured on the diagram, or, when the angle *a c b* is known, can be readily ascertained by reference to a Table of natural sines, &c.

Make *c* the centre and *b c* the radius; form the arc *b e*, then draw the line *e f* at right angles to *b c*. The distance *b f* is the versed sine in the Table, and represents the loss in height when the horse leans forward; *e f* is the sine, and *f c* the cosine, the tabular values of which for the given angle, represent in decimals the proportionate length of these lines, with reference to the radius which measures 1·0. Thus, we have found that *b c* is 6 times as great as *a b*; and as *b c* is equal to 1·0, we divide it by 6, and get 0·166 for *e f*. On referring to the Table, we find that this is the sine of 9° 33, the versed sine of which is ·01385, represented by *b f*, and the cosine ·9861 represented by *f c*.

We have here made *e f* and *f c* take the place of *a b* and *b c* in the 1 to 6 proportions of the sine and radius, because, as we shall by-and-by explain fully, the three

sides of the triangle *a b c* are in the same relative proportion to one another as those of the lesser triangle *e f c*, the angles being the same in both; and, as the values of the sides of a triangle are ruled by the angles, whatever be the size of the triangular figure, these values would be the same were we to employ the reduced triangular figure *h i c*, in which case *b c* would be to *i c* as *a b* to *h i*.

23. A heavy horse of less muscular strength than one lighter, can bring more pressure to bear upon the breast-band, and therefore has an advantage over the lighter animal. A horse may project its weight not against a breast-band, but against a collar resting on the shoulders. This does not affect the principle we are explaining, and the band forms a simpler illustration.

24. To start a load suddenly requires much more power than is needed to maintain uniform motion afterwards; and the same may be observed regarding a sudden increase of speed; because the resistance of the inertia of the load is here brought into action, in addition to the nearly uniform resistance due to friction.

We will leave friction out of account, and, with the help of Fig. 5, will explain the cause of this increased

Fig. 5.

resistance. We will assume that the speed on the lower line *e c* is equal to 264 feet per minute; and that the speed on the upper line *a b* is 352 feet per minute; and that the load is 2500 lbs. weight; and will assume further, that the horse is exerting a power of 125 lbs. in pulling this load at the rate of 264 feet per minute on the level.

We wish to carry the speed suddenly up to the higher velocity, say in a distance of 20 feet along the line *e b*. We represent *e b* as an inclined plane, merely to exhibit in sensible form the rise in speed; because the simple

rules applicable to the incline plane, do not apply here to the inertia of matter.

25. Resistance to motion, of a body at rest and free to move, when friction is excluded, is the force of inertia. It operates equally in a start from a state of rest, as in a sudden increase of motion.

26. The measure of time usually employed in computing the measure of the resistance is 1 second; and the velocity g acquired at the end of 1 second, by a body falling freely, is employed as a "constant" divisor of the weight to be moved, to get what is technically termed, the "Mass."

Thus, let W = the weight; g = gravity; and M = the mass; then $\frac{W}{g} = M$.

The force of gravity is here employed, because it is a known fixed quantity for uniformly accelerated free motion through space.

A rolling body has not absolutely free motion, because of the friction of the surface it rolls upon; but, as the amount of this friction is independently found, and taken into calculation; and, as what is termed the momentum of a body in motion is ruled by the velocity of the weight, the known quantity "g," for free motion, is employed as the standard for determining the precise value of the rolling velocity. It represents a velocity of 32⅙ feet at the end of 1 second, when let fall from a state of rest; and is at the same time, the velocity imparted to a falling body, or taken from a rising body, by gravity, during each second of its free motion, in addition to, or subtraction from, the velocity of each preceding second. We will presently explain this at greater length. (Paragraph 39.)

The equation, put into arithmetical form, stands thus:

$$\frac{W}{g} = M = \frac{2500}{32 \cdot 166} = 77 \cdot 72.$$

27. To make this result of use, we must now divide

the velocity in feet per second, of the body in motion, by the time occupied in the passage from *e* to *b*; then, employ the quotient to multiply the quantity represented by M. We thereby get the resistance of the inertia, expressed in pounds.

28. The velocity assumed here must be the average between the speeds at *e* and *b*; that is, $264 + 352 = 616 \div 2 = 308$ feet per minute, or 5·13 feet per second; and, as we assume that the change to the 352 feet speed is effected in the distance of 20 feet, we find that $308 \div 20 = \frac{1}{15 \cdot 4}$ minute $= 3 \cdot 9$ seconds; so that $\frac{5 \cdot 13}{3 \cdot 9} = 1 \cdot 316 \times 77 \cdot 72 = 102 \cdot 28$ lbs. force, requiring to be expended by the horse in overcoming the inertia of the load, in the sudden increase of speed, in addition to the 125 lbs. traction for uniform velocity; thereby making the drag upon the horse, in the passage from *e* to *b*, equal to 227·28 lbs.

The full equation is thus expressed,—

$$\frac{W}{g} \times \frac{v}{t} = M \times \frac{v}{t} = \text{resistance};$$

v = velocity in feet per second; t = the time in seconds occupied in the passage.

The 227·28 lbs. strain thus found, we now multiply by the 308 feet per minute average speed, and thereby get 70,000 work units; which, when divided by 32032, the unit standard for the speed of 308 feet per minute, as in Table 1, gives 2·18 horse-power.

$$227 \cdot 28 \times 308 = 70000 \div 32032 = 2 \cdot 18 \text{ H.P.}$$

We here see that in the drag from *e* to *b*, during 3·9 seconds, the horse has been expending fully two and a sixth times as much force as was required of it at the lower speed along the line *e c*; consequently, were this strain continued so as to extract a day's work, that day's

work would be completed in less than half the time the animal would continue at work at the lower speed.

29. We learn from Bevan's notes in the *Philosophical Magazine*, that the mean force of good horses engaged in ploughing, is about 160 lbs., at the rate of $2\frac{1}{2}$ miles an hour; doing full day's work continuously for many weeks without injury. But, as the strain in ploughing is steady, whereas, in ordinary cart draught, it is of a jolting nature, we will take 8 hours as the term of a day's work, with 125 lbs. pull at 264 feet per minute; and, dividing the 8 hours by the 2·18 horse-power, get 3·67 hours as the duration of a day's work, when drawing 227·28 lbs. at the *e* to *b* average rate of 308 feet per minute.

$$\frac{8}{2{\cdot}18} = 3{\cdot}67 \text{ hours.}$$

30. We will now assume that the horse is allowed a length of 50 feet, to increase the speed from 264 to 352 feet per minute.

The average speed on the line *e a* is the same as for *e b*, viz., 308 feet per minute, or 5·13 feet per second; but, as the distance *e a* is greater than *e b*, so must the total time occupied be greater; thus

$$308 \div 50 = \frac{1}{6{\cdot}16} \text{ minute} = 9{\cdot}74 \text{ seconds.}$$

Employing the same formula as for the *e b* distance, we have $\left(\frac{W}{g} = M\right) = \left(\frac{2500}{32{\cdot}166} = 77{\cdot}72\right)$ as before; but for $\frac{v}{t}$ we have $\frac{5{\cdot}13}{9{\cdot}74} = {\cdot}527$, by which we multiply M, and get $77{\cdot}72 \times {\cdot}527 = 40{\cdot}95$ lbs. additional force required to overcome the inertia of the load on the line *e a*, so that $125 + 40{\cdot}95 = 165{\cdot}95$ lbs. total strain. And further,

$165{\cdot}95 \times 308 = 51112$ units of work per minute,

$\frac{51112}{32032} = 1{\cdot}596$ horse-power.

$\frac{8 \text{ hours}}{1 \cdot 596} = 5 \cdot 01$ hours duration of day's work at the *e a* rate of increase, with a strain of 165·95 lbs. when the horse cannot work without injury, longer than 8 hours per day, with a strain of 125 lbs.

31. We here see that, with a given weight of load, and velocity of motion, the intensity of the strain upon the horse is ruled by the time allowed; that is, by the term "*t*" in the equation.

Were only 1 second allowed, the equation would stand thus: $77 \cdot 72 \times \frac{5 \cdot 13}{1} = 398 \cdot 7$ lbs. force required to overcome the inertia of the load, in addition to the 125 lbs. uniform pulling strain; and the line *e b* in this case would be as 5·13 to 1 for the rise *d b*.

$398 \cdot 7 + 125 = 523 \cdot 7$ lbs. total strain upon the horse; so that, supposing the horse weighs 1500 lbs., it would have to throw fully one-third of its weight upon the collar or breast-band, Fig. 4, which would cause the body to project so far in advance of the feet, and, at the same time so reduce the weight upon which the frictional adhesion of the feet upon the ground is dependent, that the animal would be more likely to fall, than to give motion to the load.

32. The ultimate speed on reaching the higher level *a b*, is greater than the average speed on the rise to it; consequently, we might expect that the traction of the load, multiplied by the speed, would here place more strain upon the horse, and therefore sooner exhaust it; but, as the speed will now be uniform, and, as the inertia or deadness of the load is unfelt either in a state of rest or of uniform motion, we are free, in the case of a wheeled carriage, to ascertain the number of units of work done, simply, by multiplying the frictional resistance on road and axles, represented by the pull, by the distance in feet travelled; then, dividing this number by 33000, we get

the horse-power expended. Thus: 125 lbs. × 352 feet = 44000 units, ÷ 33000 = 1·33 nominal horse-power as the rate of the effort. But, employing the 29216 unit measure of Table 2, we get 44000 ÷ 29216 = 1·5 actual horse-power, supposing the labour to be continued for the same number of hours as constitutes a working day at the lower speed of 264 feet per minute.

33. At the higher speed, with animal power, the day's work is of shorter duration than for the lower speed; but in a less simple ratio than for nominal horse-power. We will explain this with the help of Tredgold's Tables.

We employ the tabular values of average strength more with a view to find the comparative decrease of pulling force as the speed increases, than to fix positively the quantities of pulling force, or the precise time of a day's work for a horse; because the average must necessarily be in the middle of so wide a range from weakest to strongest, as to be of indefinite practical application.

Employing Tredgold's Table (2), with the pull constant at 83 lbs., we give in Table (3), the units per minute, and the total number that constitutes a day's work at the different velocities.

Miles.	Feet.	lbs.	Units. Minute.	Hours.	Minutes.	Day's work units.
$2\frac{1}{2}$	......220 ×	83 =	18260 ×	11·5	× 60 =	12599400
3	......264 ×	,, =	21912 ×	8·0	× ,, =	10517760
$3\frac{1}{2}$	......308 ×	,, =	25564 ×	5·9	× ,, =	9049620
4	......352 ×	,, =	29216 ×	4·5	× ,, =	7888320
5	......440 ×	,, =	36520 ×	2·9	× ,, =	6354480
6	......528 ×	,, =	43824 ×	2·0	× ,, =	5258880
7	......616 ×	,, =	51128 ×	1·5	× ,, =	4601520
8	......704 ×	,, =	58432 ×	1·12	× ,, =	3926580
9	......792 ×	,, =	65736 ×	0·90	× ,, =	3549720
10	......880 ×	,, =	73040 ×	0·75	× ,, =	3286800

34. The weight being constant, the minute units increase simply with the speed; thus, 10 miles divided by $2\frac{1}{2}$ miles

give 4; and 73040 minute units divided by 18260, give likewise 4 times as much work done in carrying a strain of 83 lbs. the distance of 880 feet, as in carrying it only 220 feet.

35. The horse, however, having to go with 4 times the velocity, has only, approximately, about one-fourth of the pulling power, that it has at the lower velocity; so that, having to do 4 times as much work, with its pulling strength about 4 times as weak, we find that, at the 10 mile speed, the strain is nearly $4 \times 4 = 16$ times as great as it has strength for; so that the time of its endurance, without injury, is only about $\frac{1}{16}$th of the time for the $2\frac{1}{2}$ mile rate.

36. We see that, according to Table (3), the number of units in the day's work decreases as the speed increases; whereas, the number of units in the minute's work increases.

The decrease in the day's work is ruled by the decrease in the strength of the horse at the higher speeds, represented by the decrease in the time; thus, $12{,}599{,}400 \div 3{,}286{,}800 = 3{\cdot}8$ times less work at the 10 mile rate, than when the rate is $2\frac{1}{2}$ miles. We now multiply this by the 4 got by dividing 880 feet, by 220 feet speed, and thereby get 15·2, in place of the approximate 16.

Dividing the 11·5 hours of the $2\frac{1}{2}$ miles by 15·2, we get ·75 hour, or 45 minutes' duration of the day's work when drawing 83 lbs. at the rate of 10 miles an hour.

Employing the quantities given in Table (1), we find similarly for strength at the $2\frac{1}{2}$ and the 5 miles; thus, $5 \div 2{\cdot}5 = 2$ for speed; and $32120 \div 18330 = 1{\cdot}75$ for work; and $2 \times 1{\cdot}75 = 3{\cdot}504$ for weakness; so that, $146 \div 3{\cdot}504 = 41{\cdot}66$ lbs. pulling strength of the horse at 5 miles, when its pulling strength at $2\frac{1}{2}$ miles is 146 lbs.

37. We shall now, without direct reference to units,

employ the gross load in tons, and the speed in miles, of Table (2), to get the duration of a day's work in relation to the 8 hours of the 3 mile speed. We thereby get the same results with fewer figures, than for Table (3).

The load at 3 miles = 92 tons
,, ,, 5 ,, = 57 ,,
,, ,, 10 ,, = 28·8 ,,

For the 5 miles, we get, 92 ÷ 57 = 1·614 times for power; and 5 ÷ 3 = 1·66 times for speed; so that we have 1·614 × 1·66 = 2·679, by which we divide the standard 8 hours, and get 2·98 hours' duration of the day's work at the 5 mile speed.

For the 10 miles, we get, 92 ÷ 28·8 = 3·194 for power; and 10 ÷ 3 = 3·33 for speed.

Multiplying these two quantities together, we get, 3·194 × 3·33 = 10·636, by which we divide the 8 hours and get ·75 hour, or 45 minutes.

38. These results show that it is sufficiently near for approximate estimate in practice on the road, to say that the pulling strength of a horse is inversely as the square of the velocity; so that, when we know the load, and speed, and distance, that compose the day's work for any particular horse, and wish it to move faster, the square of the number of times the lower speed is contained in the higher, gives a quantity by which to divide either the load, or the time, for the day's work at the higher speed.

SECTION III.

39. The force of gravity, represented by the term "g," in the equation $\frac{W}{g} = M$, which we used in paragraph 26 when determining the force of inertia, is employed only

where speed is uniformly accelerated or retarded, as in bodies falling or rising freely through space; or, in rolling or sliding freely down incline planes; or, increasing or slackening speed on level roads; but it is not necessary where the power and speed are uniform, when we wish to know only the consumption of power during the uniform action; because, when the speed is established, the power has no other resistance opposed to it than that of friction.

40. Should we, however, seek to ascertain the amount of force stored up in the body in motion, the force of gravity *g*, is present in uniform velocities as well as in velocities that are still under accelerating impulse.

Thus, if we suppose the force to be rendered sensible by a blow, in which the motion would be suddenly stopped, the effect is the same when the blow is made the instant the uniform rate of speed is reached as when that speed has been maintained an hour; because, the force of the blow is simply equal to the force accumulated in the body by the succession of impulses that produce the accelerated motion of its rise from a state of rest to the speed at the moment of striking; and, the moment the velocity becomes uniform, this succession of impulses terminates; but their effect remains, and cannot be increased, without an increase of velocity; and can be diminished only by a reduction of velocity. (Paragraph 55.)

We have already explained the simple value of *g*, paragraph 26, so far as concerns Fig. 5; but fuller explanation is necessary to make the question of accumulated force clear.

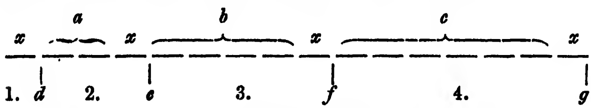
41. Let us take the first second of a fall from a state of rest. At the end of the second, the body is falling at the rate of $32\frac{1}{6}$ feet per second, but, it has fallen a distance of only one-half this, or $16\frac{1}{12}$ feet.

Into the next second it has carried the $32\frac{1}{6}$ feet velocity, and has acquired at the end of it a double velocity of 2 ×

$32\frac{1}{6} = 64\frac{1}{3}$ feet per second; and, carrying this latter velocity into the third second, has acquired at the end of it a three-fold velocity of $3 \times 32\frac{1}{6} = 96\frac{1}{2}$ per second; and, carrying this into the fourth second, has acquired at the end of it a velocity of $4 \times 32\frac{1}{6} = 128\frac{2}{3}$ feet per second, and so on; increasing the velocity of descent by $32\frac{1}{6}$ feet per second.

42. The distance fallen, however, for each successive second taken separately, is, for the first second, $16\frac{1}{12}$ feet; for the next, $16\frac{1}{12} + 32\frac{1}{6} = 48\frac{3}{12}$ feet; for the third second, $16\frac{1}{12} + 64\frac{1}{3} = 80\frac{5}{12}$ feet; and, for the fourth second, $16\frac{1}{12} + 96\frac{1}{2} = 112\frac{7}{12}$ feet; and so on. In each successive second, the distance additional to the velocity introduced from each preceding second is $16\frac{1}{12}$ feet; so that, at the end of the fourth second, the total distance fallen is, $16\frac{1}{12} + 48\frac{3}{12} + 80\frac{5}{12} + 112\frac{7}{12} = 257\frac{4}{12}$ feet.

To make the matter more clear, we will present it in simpler form, by means of spaced lines; thus,—



d	= distance fallen in		1	second.
$d + e$	= ,,	,,	2	,,
$d + e + f$	= ,,	,,	3	,,
$d + e + f + g$	= ,,	,,	4	,,

a = velocity carried from d into $d\ e$.
b = ,, ,, e ,, $e\ f$.
c = ,, ,, f ,, $f\ g$.

Each of the smaller spaces, as x, represents a distance of $16\frac{1}{12}$ feet.

	Spaces.	Spaces.	Feet.	
d..........	$= 1$	$= 1 \times 16\frac{1}{12}$	$= 16\frac{1}{12}$	feet.
$d + e$.......	$= 1 + 3$.......	$= 4 \times$,,	$= 64\frac{1}{3}$	,,
$d + e + f$...	$= 1 + 3 + 5$....	$= 9 \times$,,	$= 144\frac{9}{12}$	,,
$d + e + f + g$	$= 1 + 3 + 5 + 7$	$= 16 \times$,,	$= 257\frac{4}{12}$	,,

		Spaces.	
$a =$ velocity	$= 1 \times 2 =$	$2 \times 16\frac{1}{12}$	$= 32\frac{1}{6}$
$b =$,,	$= 2 \times 2 =$	$4 \times$,,	$= 64\frac{1}{3}$
$c =$,,	$= 3 \times 2 =$	$6 \times$,,	$=$

The $16\frac{1}{12}$ feet spaces lettered x, represent the successive distances that generate the $32\frac{1}{6}$ feet velocity additional for each successive second; hence, the accumulative increase of the bracketed intervals a, b, and c by $2 \times 16\frac{1}{12} = 32\frac{1}{6}$, for each second.

To make the distances x apparent, it is necessary to show them detached from the spaces covered by the introduced velocities, a, b, and c; but, it must be understood, that the force of gravity due to the fall x, operates from the beginning to the end of the second of time that it belongs to, and is in nowise affected by being mixed up with the velocities generated in preceding seconds.

43. When we square the times d, e, f, and g, we find that the square number just equals the number of times the $16\frac{1}{12}$ feet distance is contained in the total distance fallen; thus,—

		Square of time.	$16\frac{1}{12}$ feet spaces.
d^2	$= 1^2 =$	1	$= 1$
$(d + e)^2$........	$= 2^2 =$	4	$= 1 + 3$
$(d + e + f)^2$....	$= 3^2 =$	9	$= 1 + 3 + 5$
$(d + e + f + g)^2$	$= 4^2 =$	16	$= 1 + 3 + 5 + 7$

44. When we square the velocities a, b, and c, we get the same ratios, when the square of a is represented by 1; thus,—

Seconds.		Velocity.		Ratio.
$1^s = a =$		$32\frac{1}{6}^2 =$	$1024 =$	1
$2 = b =$		$64\frac{1}{3}^2 =$	$4096 =$	4
$3 = c =$		$96\frac{3}{6}^2 =$	$9216 =$	9
$4 =$		$= 128\frac{4}{6}^2 =$	$16384 =$	16

45. The ratios for time and velocity are the same when thus treated; but, as the square of a whole quantity is

4 times as great as the square of half that quantity, so is the square of the velocity $32\frac{1}{6}$ feet, 4 times as much as the square of the $16\frac{1}{12}$ feet fall required to generate that velocity; hence, when we wish to ascertain the height of the fall required to impart a given velocity, we divide the square of the velocity by $4 \times 16\frac{1}{12} = 64\frac{1}{3}$, that is, by $2g = 2 \times 32\frac{1}{6} = 64\frac{1}{3}$.

46. As already indicated, the total distance fallen from the state of rest is found quickly, by multiplying the $16\frac{1}{12}$ feet fall of the first second by the square of the number of seconds that form the total time; thus, $T^2 \times \frac{1}{2}g = \text{Time}^2 \times 16\frac{1}{12} =$ fall in feet.

Seconds.	Square.		Feet.	
$1^2 =$	$1 \times 16\frac{1}{12} =$		$16\frac{1}{12}$ feet fall.	
$2^2 =$	$4 \times$	,, $=$	$64\frac{4}{12}$	,, ,,
$3^2 =$	$9 \times$	,, $=$	$144\frac{9}{12}$	,, ,,
$4^2 =$	$16 \times$	,, $=$	$257\frac{4}{12}$	,, ,,

47. And, when we know the velocity, and wish to ascertain the height from which a body must fall to acquire it, we square the velocity and divide by twice g; thus: $\dfrac{\text{Velocity}^2}{2g} =$ fall in feet.

Seconds.

$$1 \ldots\ldots\ldots \frac{32\frac{1}{6}^2}{2 \times 32\frac{1}{6}} = 16 \text{ feet}\tfrac{1}{12} \text{ fall.}$$

$$2 \ldots\ldots\ldots \frac{64\frac{1}{3}^2}{2 \times 32\frac{1}{6}} = 64\tfrac{4}{12} \quad ,, \quad ,,$$

$$3 \ldots\ldots\ldots \frac{96\frac{1}{2}}{2 \times 32\frac{1}{6}} = 144\tfrac{9}{12} \quad ,, \quad ,,$$

$$4 \ldots\ldots\ldots \frac{128\frac{2}{3}}{2 \times 32\frac{1}{6}} = 257\tfrac{4}{12} \quad ,, \quad ,,$$

48. When the terminal velocity is known, and the time required to give it is wanted, we divide the velocity by $32\frac{1}{6}$ for 1 second, and thereby get the number of seconds; thus, for the velocity of 352 feet per second; $\dfrac{352}{32{\cdot}166} =$ 10·94 seconds.

49. When the height of the fall is known, and we require to know the time, we divide the height in feet by $16\frac{1}{12}$, and the square root of the quotient gives the number of seconds; thus, for a fall of 160 feet: $\frac{160}{16 \cdot 08} = 9 \cdot 95$, which is the square of the number of seconds; so that, $\sqrt{9 \cdot 95} = 3 \cdot 15$ seconds, time occupied in falling.

50. The units of work accumulated in a falling body are ascertained by finding the height of fall required to impart the given velocity, and multiplying that height by the weight of the body; because, the same force is expended in rising to that height, as is acquired in falling from it. (Paragraph 54.)

51. Thus, a rifle-ball, fired straight up into the air, has its velocity diminished at the same accelerated rate as it has it increased when falling freely; because, the attraction of the earth is a constant quantity, without regard to the direction of the motion; and acts equally in high velocities as in low. When the motion is from earth, it acts by abstracting from the upward impulse an amount per second equal to the additional impulse per second that it generates in motion towards earth; so that, employing the space-line figure, paragraph 42, the ball, starting to rise, say from *g* to *d*, with the velocity due to *g*, will cross the successive points, *f*, *e*, and *d*, at the same equal intervals of time as in falling; and, on passing *d*, it will occupy the same time, viz., 1 second, in reaching a state of rest at the extremity of the $16\frac{1}{12}$ feet beyond *d*, that it occupied in rising from *g* to *f*; and, further, on starting downward from this point of rest beyond *d*, it will re-pass the points *d*, *e*, *f*, and *g*, in the same equal intervals of time it occupied in rising; so that, the force expended in the rise can only be equal to the force acquired in the fall; but, the half or mean of the time occupied in the whole fall is at the point *e*, where the velocity is as 2 to 4 for the velocity at *g*; and the dis-

tance $d + e$ is as 1 to 4 for $d + g$, these ratios being constant expressions whether d, e, f, and g represent whole seconds or fractions of seconds.

52. In starting from a state of rest, the velocity at the middle of the *time* occupied in falling can be no more than *one-half* the rate acquired at the end of the whole time; thus, for 1 second it is 32; for 2 seconds 64; and for 4 seconds 128 feet.

53. The *distance* fallen, however, by the body in this half time, is only *one-fourth* of the distance for the whole time: thus, for 1 second it is 16; for 2 seconds 64; and for 4 seconds 257 feet, roughly.

As regards the velocity, the half rate is owing to the velocity increasing simply by regular additions of 32 feet per second; so that one-half the time can contain only one-half the number of these additions. Whereas, as regards total distance fallen, the distance accumulates at the rate of 4 times for every doubling of the time, which is equivalent to one-fourth for half time. We therefore have here the mean-time distance as $\frac{1}{4}$ to $\frac{1}{2}$ for the mean-time velocity, and, as the work stored in the falling body is as the total height fallen, we employ $2\,g$ when estimating the work by the velocity.

In treating this one-fourth power in paragraph 45, the conditions were different, because we there squared the velocity, whereas here we are treating of simple additions and straight distances; but the $2\,g$, in relation to the squared velocity, is a divisor equal to a power of 4, so that it gives the result here wanted, because the velocities square in the same ratio as the seconds of time, and the $2 \times 32 = 64$, for the divisor of the feet velocity squared, is equivalent to the fourth of 64; that is, 16 feet fall for a multiplier of the units of time squared.

54. In finding the consumption of power, when motion is uniform, we multiply the speed in feet per minute by the weight of the moving force in pounds, and thereby get

the number of units of work per minute; whereas, in finding the force that has been expended upon, and therefore accumulated or stored up in the body, in raising the motion from a state of rest, to any given velocity, we multiply the square of the velocity in feet per second by the whole weight of the body, and divide by $2g$; thus

$$\frac{V^2 \times W}{2g} = \text{work units.}$$

Applying the formula to the carriage load of 2500 lbs., with a speed of 352 feet per minute, or 5·866 feet per second, we have the question thus expressed arithmetically:

$$\frac{5{\cdot}866^2 \times 2500}{2 \times 32\frac{1}{6}} = 1337$$ units of work accumulated in the body on attaining the velocity of 5·866 feet per second; and this store of work remains constant for that velocity. Or, as expressed in paragraph 50, the distance fallen to produce the 5·866 velocity is ·5333 feet (paragraph 47), and 2500 lbs. × ·5333 = 1337 units.

55. Should the pulling power be reduced to a degree that would maintain a uniform motion of, say, only 4 feet per second, the body would have to discharge 715 units of this work stored up in it before it could assume the lower uniform speed of 4 feet per second; and could discharge them only against the frictional resistance of the surfaces it moved in contact with. And, as friction is nearly a constant quantity per foot of motion, dependent upon the nature of the materials, the consumption of power in friction, in a given *time*, is so much greater at the higher speed than at the lower, that, when the power is suddenly reduced, the moving body has to supply the difference from the impulse stored up within itself, or else stop; and, that it does supply it is evident, from reduction of speed being so far gradual, that, in the case of horse-draught, the carriage tends to run forward upon the horse, when the horse slows too abruptly. Were there no im-

pulsive power stored up within the body in motion, the motion would cease as abruptly as the pull.

For higher velocities, the resistance of the air consumes a portion of the liberated impulse.

$$\frac{4^2 \times 2500}{2 \times 32\frac{1}{6}} = 622 \text{ units of work,}$$

and $1337 - 622 = 715$ units difference.

56. Now, we must here observe that these amounts of work stored, represent quantity only, and not intensity. The motion that accumulated them may have occupied 1 minute, or 1 hour; so that, before we can properly estimate their practical values, as regards either the power expended in their accumulation or their effect in overcoming resistance opposed to their motion, we must ascertain the time occupied.

The formula employed to find the force required to overcome inertia is here applicable. (Paragraph 28.)

$$\frac{W}{g} \times \frac{v}{t}$$

57. We will assume that the motion of 5·866 feet per second is arrested in 1 second, and that it is arrested by running up against a strong spiral spring, which must be long enough to allow of closing to the extent of at least 5·866 feet: then

$$\frac{2500}{32\frac{1}{6}} \times \frac{5{\cdot}866}{1{\cdot}0} = 455{\cdot}9 \text{ lbs. weight, that would be}$$

registered in the compression of the spring.

Were the stoppage more of the nature of a blow, so as to occupy, say $\frac{1}{60}$th of a second, we would get

$$\frac{2500}{32\frac{1}{6}} \times \frac{5{\cdot}866}{0{\cdot}016} = 27435 \text{ lbs. force that the spring would}$$

register, and this is just 60 times the force when the time allowed is 1 second.

Supposing that the velocity were at the rate of 128 feet per second, due to a fall of 4 seconds' time; and that the

time occupied in stopping were 1 second; and that the spring consequently shortens its length by 128 feet. We have the velocity decreasing from the beginning till the end of the time allowed for stoppage, at a uniformly retarded rate, as in the case of the rifle-ball fired from *g* to *d*, paragraph 51, and the 128 feet of spring might be marked with similarly proportioned intervals, *g*, *f*, *e*, and *d*, to mark the distance: but, as the ball occupies 4 seconds in expending its free impulse, whereas, here, as against a spring, it is allowed only 1 second, we have ¼ seconds taking the place of the whole seconds of free motion, so that, the times of *g f*, *f e*, *e d*, and *d r*, would be only ¼ the times for free motion; and, as each successive ¼ second of the 1 second stoppage destroys as much of the velocity as each successive second from *g* to *d* of the 4 seconds, we have the intensity of the resistance in the 1 second stoppage 4 times as great as the intensity of the 4 seconds; and this explains why the $\frac{1}{60}$th second stoppage gives 60 times the intensity of the 1 second stoppage.

58. There is no body so void of elasticity, that, in a collision between two, the force could be communicated without a certain amount of compression of their surfaces in contact; and the time occupied in effecting this compression would take the place of "*t*" in the equation.

SECTION IV.

59. The Laws relating to the motion of bodies on incline planes apply equally to rolling and to sliding motion; but, as our present inquiry has reference mainly to the traction of rolling loads, our further observations here will apply to rolling motion only.

When the road is on an inclined plane, the downward tendency, considered apart from friction, is found by dividing the total weight by the number of times the rise is contained in the length of the slope. We will explain this by means of Fig. 6, in which we exaggerate the dimensions for greater clearness.

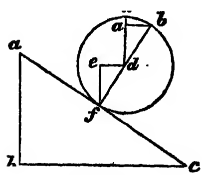


Fig. 6.

Let the rise *a b* equal 1 foot, and the slope *a c* equal 20 feet, and the load borne by the wheel *w* equal 2500 lbs., as in the case of Fig. 5, and let the horse be pulling up-hill.

60. We have explained how the horse throws his weight forward in advance of his feet, in order to get the required pressure upon the collar.

The load upon the wheel in Fig. 6 exerts its backward pressure in a similar manner, thus: From *d*, the centre of the wheel, draw a line *d f* at right angles to the slope *a c*; then draw a line from *f* to *e*, parallel to *a b*, and a line from *d* to *e* parallel to *b c*.

We have now the angle *d e f* equal to the angle *a b c*, and the other two angles at *d* and *f* respectively equal to the angles *a* and *c*, in the order we here name them. The triangle *d e f* is by construction simply a small-sized duplicate of the triangle *a b c*, in which the side *d e* represents the rise *a b*; the side *e f* represents the horizontal line *b c*; and the side *d f* the slope *a c*.

61. We see that the wheel is supported on the slope at the point *f*, and as the line *a b* is at right angles to the horizontal line *b c*, it follows that *e f*, which is parallel to *a b*, is also at right angles to *b c*, so that *e* is plumb with the point of support *f*, consequently, when the length of the line *d f*, corresponding with *a c* of the slope, is made to represent the full weight of the load, the length of the line *d e*, corresponding with the rise *a b*, represents the proportion of the load which is exerting a backward pull,

because the centre of gravity *d* of the whole load overhangs *f*, the point of support, to the extent measured by the line *e d*; so that, when *d e* and *a b* are respectively as 1 to 20 for *d f* and *a c*, the load has been thrown out of balance to the extent of one-twentieth of its weight, and this is the measure of the backward pull, $\frac{2500}{20} = 125$ lbs.

62. The length of the line *e f*, corresponding with *b c*, gives the proportionate pressure of the load upon the surface of the slope.

We can ascertain the vertical pressure when we know the length of the slope *a c*, and corresponding horizontal distance *b c*, by multiplying the total load by the horizontal distance *b c*, and dividing the product by the distance *a c*; or, when we know the angle *a c b*, multiplying the load by the cosine *b c*, and dividing by the radius *a c*. Thus, for a gradient of 1 to 20, the angle *a c b* is about 2° 52, the cosine *b c* of which is ·9987, so that

$\frac{\cdot 9987 \times 2500}{1 \cdot 00} = 2496 \cdot 7$ lbs., the vertical pressure of the load at *f*. To find the backward pull by the angle, we multiply the load by the sine of the angle, thus: $\cdot 050 \times 2500 = 125$ lbs.

63. As regards the inclination of the traces by which the horse is yoked to a carriage, we will suppose them attached to the centre of the wheel in Fig. 7, so that the radius of the wheel may represent the radius of our tabular values.

Were the traces leading in the direction of the horizontal line *a b*, and the pulling force just very slightly more than equal to the resistance to motion of the wheel on a smooth surface, the pulling force would be represented by the horizontal radius *a e*, and the wheel resistance by the vertical radius *a j*; but, supposing the traces were to lead in the direction *a c*, we would have the horizontal pull re-

presented by the reduced length *a f*, which is the cosine of the angle *e a h*.

The whole pulling force would be in amount the same as before, and would be represented by the radius *a h*; but as its direction is upward, a certain portion is exerted in supporting the weight of the wheel, and that portion is represented by the versed sine *e f*, so that, supposing the

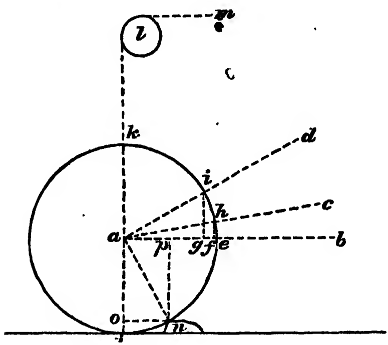


Fig. 7.

angle *e a h* equal to 10°, the versed sine of which is ·0151, and the cosine ·98480, these decimal parts of the unit 1·00 for the whole pulling force exerted, respectively represent the horizontal and the vertical effects of that force in the line *a c*; so that when added together, they just equal the radius for whole force *a h* or *a e*, thus: ·0151 + ·98480 = 1·00.

64. Let the pulling force be equal to 100 lbs. This, when multiplied by the versed sine ·0151, gives 1·51 lbs. lost in the vertical direction.

Were the angle equal to 30° *d a b*, the versed sine of which is ·1339, the loss would be ·1339 × 100 lbs. = 13·39 lbs., represented by *g e*; and were the angle 90° *k a e*, with the traces leading over the pulley *l*, the loss of force in the horizontal direction would be total, because

the versed sine has here become equal to the radius $a\ e$, and the power is now altogether exerted in the vertical direction $a\ k$.

65. The upward inclination of the traces is of service mainly on rough roads, where the wheels have to surmount hillocks and small stones.

In explaining the service it here renders, we will still employ Fig. 7.

Let n be a hillock, and let the angle $e\ a\ n$ be 60°, the sine $p\ n$ of which is ·8660; and the cosine $p\ a$, ·500; and let the weight of the wheel equal 500 lbs.

Now as the weight of the wheel has its centre of gravity at a, it is clear that, when the point of resistance is at n, the leverage of the overhanging load is represented by $p\ a$, the cosine of the angle; and it is equally clear that the leverage of the horse-power is represented by the sine $p\ n$ when the pull is horizontal; that is, the horizontal force required to carry the load at a, over n, is as $p\ a$ to $p\ n$. So that, as the load is 500 lbs., we make the question one of simple proportion; thus,—

$p\ n : p\ a :: 500 :$

·866 : ·50 :: 500 : 289 lbs., when the pull is in the direction $a\ b$.

But, when the pull is in the direction $a\ d$, we have the angle $e\ a\ n + e\ a\ i = 60° + 30° = 90°$; and, as this is a right angle, we have the sine equal to the radius, so that the power is acting with the full leverage of the radius $a\ n$; hence, as the load at a has to be lifted, we have the resistance of the load acting as $p\ a$ or $n\ o$, to $a\ n$; and, as the radius $a\ n$ is represented by 1·0, we have

$a\ n : n\ o :: 500 :$

1·0 : ·50 :: 500 : 250 lbs. force required to surmount the hillock.

66. We assumed the pulling power to be only 100 lbs. for the smooth road. It has here to be raised to 2·5 times

this force, and the horse would require to exert this 250 lbs. pressure on the traces, were it starting the load from a state of rest; but, as it would never be called upon under ordinary circumstances to start directly against so great an obstruction, we must consider the hillock as met with when in motion; the momentum of the weight, greater or less according to the speed, would carry the wheel over: but, if the force of the momentum were just sufficient to overcome the resistance of the lift, the horse, when the wheel landed on the other side, would have to exert strength equal to what is required of it at a fresh start, because, the momentum being all expended in the lift, can only be restored by the exertions of the horse.

The loss which we ascertained for the angle *i a e*, when the inclination of the traces alone was in question, does not operate in the case of the angle *i a n* when that angle is 90°, but were it, say, 100°, we would then find for the loss represented by the versed sine of 10°, which is nearly equal to the loss represented by *f e* of the angle *h a e* already explained.

67. On a railway, the pull ought to be horizontal, because the way is smooth; but the point of attachment to the collar of the horse ought, for the state of rest, to be higher than the point of attachment to the waggon, so that, when the horse throws its weight forward in the act of pulling, the pulling line will not sink below the horizontal direction.

If it does sink below it, there is a loss of horizontal force equal to the value of the versed sine, and the amount lost has been transferred to the weight of the waggon, so as to make the haulage heavier by the fractional addition, in the same way as the weight upon the road or rail is lightened when the inclination is upward.

68. In the case of the hillock, the greater the diameter of the wheel, the less is the angle *o a n*, or what is in

result the same, the greater is the angle *e a n*; and, as the lessening of the angle *o a n* lessens the cosine *p a* or *n o*, which represents the leverage of the load, and increases the leverage of the power *p n* or *a o*, we have the power working at a greater advantage.

SECTION V.

69. The amount of friction upon the axle of a wheel of great diameter is equal to that upon the axle of a small wheel when the load is the same, but the horse overcomes it with less pressure in the case of the large wheel.

The pressure of the load is borne by the under surface of the axle at *i* and *t* in Figs. 8 and 9, and the axle being fixed, it follows that the wheel revolves round it as a centre, so that, on rolling on the road from *c* to *d*, or from *m* to *n*, that part which is rubbing on the axle is moved from *b* to *i*, or from *l* to *t*.

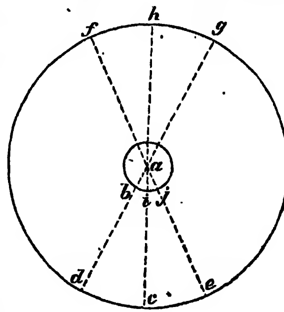


Fig. 8.

70. Now, the number of times which the distance *b i* is contained in the distance *c d*, or *l t* in the distance *m n*, gives the ratio of the leverage of the power employed in giving motion. But, as one complete revolution of the rim of the wheel at *c* gives no more than one revolution round the axle *b*, and as the circumference of a circle bears a constant relation to the diameter, we can at once use the diameters to find the leverage, and say that the fric-

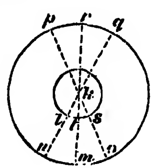


Fig. 9.

tional resistance at *b* is as much greater than the power required at *c* to overcome it, as the diameter of the wheel is greater than the diameter of the axle; hence, if the load and the diameter of the axle be alike in the two cases, but the diameter of the wheel in Fig. 8 be twice the diameter of the wheel in Fig. 9, it follows that, as the leverage in the latter case is only half of what it is in the former, twice the power will be required at *m* to overcome the friction at *l* that is required at *c* to overcome the friction at *b*.

We will explain this principle more particularly when we come to speak of pulley motion. We may here remark, however, that we speak of the power being exerted at the points *c* and *m*, because the same force is expended in the wheel, when the cart moves on the stationary road, as would be expended were the cart to remain stationary and the road to move. In the latter case of course the power would pass into the wheel at the points *c* and *m*.

71. We have already observed that in estimating horse-power, the pressure required is the measure of the power, no matter in what direction it is exerted, nor whether it be employed in lifting a suspended weight, or in overcoming frictional resistance, as in the wheel and axle of a carriage.

There are some points of difference, however, that require to be noticed, as regards the resistance of a weight hanging free over a pulley, and of a carriage-load on wheels.

We will, for simpler illustration, speak of the carriage-load as being all borne on one wheel, there being no appreciable error in so doing, because axle friction is simply as the pressure of the load, so that in shifting the whole load on to one of the four wheels, we merely, for theoretical consideration, gather into one point the friction that before was borne at four points.

We will assume that the wheel is running upon a smooth and level iron rail, as we thereby get rid of all question

about resistance from the surface of the road, and will assume, also, that the pull in both cases is horizontal, so as to leave out of account the question of the proper angle of the traces for producing maximum effect when obstructions such as stones or hillocks have to be surmounted.

72. In Fig. 10 we show a 4-wheeled carriage; as we are considering pulling force and friction simply.

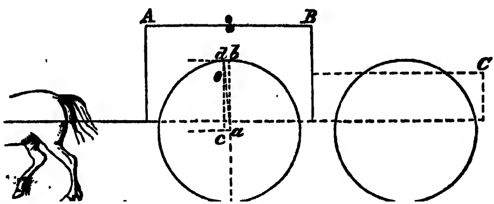


Fig. 10.

In a common cart the horse has to bear the pressure of a portion of the load upon his back, which makes the case less simple, because, firstly, we have to ascertain how much he is thus bearing—a question of mere leverage,—so as to deduct it from the axle load; and secondly, the effect of the burden as regards the firmer hold of the ground his feet are thereby enabled to take—a question relating to increased weight of body, but as this increase is not natural, the strength of the limbs is taxed more than if it were.

73. Let the load A C, in Fig. 10, equal 10,000 lbs., be contracted into the length A B, so as to bear on one wheel only, and assume, for the sake of comparison with the free load of Fig. 11, that the horse has to exert a pulling force of 150 lbs.

Rolling friction, that is, the resistance to motion experienced between the rim of the wheel and the rail it runs on, is not more than about $\frac{1}{1000}$ part of the load, which in this case would amount to 10 lbs. But we will leave that out of consideration, as our inquiry here does not

concern the strength of the horse, and will deal with the friction at the axle only, and find the point in the cir-

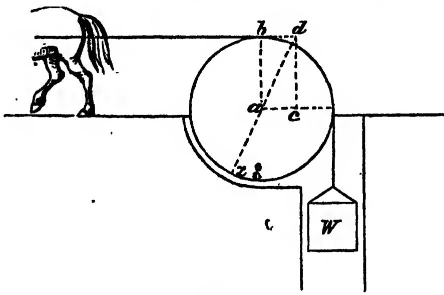


Fig. 11.

cumference of the axle where the load is bearing most severely.

74. Let the line *a b*, which we have, merely for convenience, shown equal to the radius of the wheel, represent prop ;ely the 10,000 lbs. weight, and the horizontal dista the 150 lbs. pulling force; then draw a line from *d* to *a*; this line will cut the circumference of the axle at the point of greatest friction. We may construct this diagram simply by dividing the line *a b* into $\frac{10000}{150}$ $= 66{\cdot}66$ equal parts, and making *d b* equal to 1·0 of those parts. But we may also construct it by determining the angles thus: divide the pulling force by the whole load, the answer gives the proportionate length of line *d b* for the pulling force, when the line *a b*, which represents the load, is equal to 1. Thus

$\frac{150}{10000} = {\cdot}015$, that is, the pulling force *d b* is, in this assumed case, equal to $\frac{1{\cdot}00}{{\cdot}015} = \frac{1}{66}$th part of the load.

The line *d b* is the tangent to the wheel circle; *a b* being the radius.

Referring now to a table of natural tangents, we find that ·015 is the tangent to an angle of 0° 52′; so that the angle *b a d* is only of this small amount, viz. 8 minutes, less than 1 degree.

Transferring the angle *b a d* to Fig. 12, which shows the wheel and axle enlarged, the point of greatest pressure on the axle is found to be at *c*; and as the angle serves for both axle and wheel, and *d b* is here $\frac{1}{66}$th of the radius *a b*, we find that the distance of *c* from *e* in the latter figure is $\frac{1}{66}$th of the radius *a e* of the axle; so

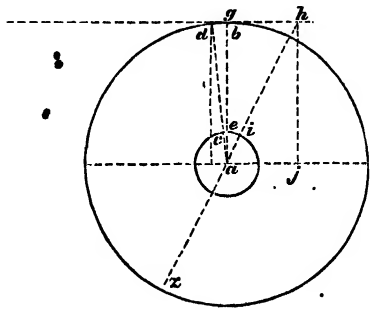


Fig. 12.

that, if the radius of the axle be 1½ inches, we have $\frac{1 \cdot 5}{66} = 0 \cdot 023$ inch, the distance of *c* from *e*, measuring horizontally. We show the angle wider than for 52 minutes, so as to make the distance *c e* apparent.

Were we to find for each axle separately with its fourth part of the total load, we would get the same angle as we have here found, because we would have to give to each fourth part of the load only a fourth part of the pulling force.

75. The horse moves at a different rate, but the work done is the same when the rope passes round the wheel to the horse along the line *b d*, Fig. 10, as when attached to the fixed axle *a*, the horizontal lead being preserved; because, as the wheel revolves round the axle *a* as a centre, *a* acts as a fulcrum at the middle of the lever *x b*, the length of which is the diameter of the wheel, so that the power at *b* has

no advantage over the resistance at x, and might as well be applied at the centre a, where there is no leverage. The resistance at x is the grip of the wheel upon the road, preventing sliding motion, and allowing the wheel to move by rolling only.

76. In estimating the frictional resistance in Fig. 10, the weight of the wheels is not included in the 10,000 lbs. load, because it is borne by and simply rolls upon the road, whereas the pressure of the carried load acts by rubbing upon the axle in just the same way as if a load of that weight with a metal sole were dragged along a smooth metal-plate greased.

In Fig. 11, however, the weight of the pulley has to be taken into account, because this weight acts here precisely in the same way as the carried load in Fig. 10, being wholly borne by the axle a.

We assume the pulling force required to raise the load w to be 150 lbs., and as part of this force is required to overcome the friction on the axle due to the weights of the pulley and the load, the load lifted cannot be equal to the pulling force. We will here ascertain how much less it is.

Let the weight of the pulley be equal to the load, which is 150 lbs. The amount of friction upon axles in motion, when oiled in the ordinary manner, is ·07 of the pressure, equal to fully $\frac{1}{14}$th, so that $150 + 150 = 300 \times \cdot 07 = 21$ lbs. weight of friction on the axle.

77. The horse, however, has the advantage of the leverage, explained with reference to Figs. 8 and 9; so that, making the radius of the pulley 18 inches, and of the axle 1 inch, we have the pulling force in Fig. 13 exerted at the outer end b, of a lever $a\,b$, which is 18 times as long as the lever $a\,c$, at the outer end of which is the frictional resistance.

With the axle fixed, the friction due to the pressure of the load is not taking effect at c, but at i. This, how-

ever, does not affect the question of leverage, as it does not matter whether the short arm $a\,c$ be doubled back upon the long arm $a\,b$, so as to lie in the line $a\,d$, or stand out as $a\,e$ or $a\,f$.

The leverage being as 18 to 1, we divide the axle friction by 18, to find the power required by the horse to overcome it. Thus, $\frac{21}{18} = 1{\cdot}15$ lbs. pressure at b, required to equal 21 lbs. frictional resistance at c or d, Fig. 13; so that $150 - 1{\cdot}5 = 148{\cdot}5$ lbs., the weight of the load that can be just balanced by the 150 lbs. pulling force.

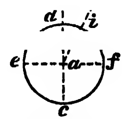


Fig. 13.

78. We require to increase the pulling force or decrease the load a very little before motion will begin; thus, if we make the load ½ lb. less, the horse will have an excess of power equal to ½ lb., which will more than enable it to move the load at the uniform pace given in Tredgold's Table (1) for a pulling strain of 150 lbs.; in fact, it would act as a slowly-accelerating force; as much slower than a free fall through space as the whole weight in motion is greater than ½ lb. (Paragraph 88.)

79. We now treat the power and the load as before to find the centre of friction on the surface of the axle, $150 + 148{\cdot}5 = 298{\cdot}5$ lbs. weight exerting vertical pressure, and 150 lbs. pulling force divided by 298·5 lbs. gives ·502 as the proportionate length of $b\,d$, Fig. 11, when the load line $a\,b$ is equal to 1.

Let us transfer these lines to Fig. 12, making $g\,h$ represent $b\,d$, and $g\,a$ represent $a\,b$, of Fig. 11. Then,

for the reasons already given in reference to the carriage forces illustrated by Fig. 12, this fraction ·502 of the radius of the axle, which is 2 inches in diameter, or 1 inch radius, will place the centre of friction on the axle at the point i, which is ½ inch from the line $g\ a$, measured horizontally, and $g\ h$ = ·502 being the tangent to a circle of which $g\ a$ is the radius, we find, on reference to the Table of Natural Tangents, that $h\ a$ is sloped at an angle of 27°, nearly, to the vertical line of $g\ a$. Any strut, therefore, set up to resist the pressure on the pulley-framing, ought to lie at this angle; that is, in the direction of the line $a\ z$, Figs. 11 and 12.

SECTION VI.

80. In Fig. 14, two men of equal weight are shown suspended by a rope from opposite sides of a pulley.

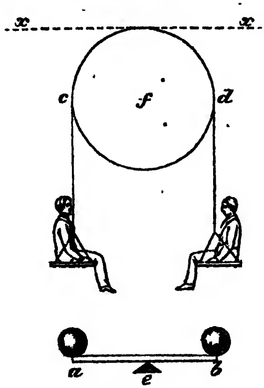


Fig. 14.

As they are of equal weight, they will remain simply balanced, like the corresponding weights which we show beneath them balanced at the opposite ends of the lever $a\ b$, the length of which is equal to the diameter of the pulley, and the fulcrum e is in the middle, to correspond with f, the centre of the pulley.

Let the weight of each man be 140 lbs., the rope has no more than this weight to bear, but the pulley f, and the fulcrum e, have 140 + 140 = 280 lbs. load to support.

The same strain results to the rope—that is 140 lbs.,

—whether it be tied to the pulley at the points *d* and *c* or passes round to be supported by the balancing weight of the other side.

81. In Fig. 15 let the diameter of the pulley *b* be 4 feet, and of the pulley *a*, 1 foot, and the weight of the man and his seat 140 lbs. We wish to know the weight at *b* that will balance his weight at *a*.

The diameter of the pulley *b*, being as 4 to 1 of the pulley *a*, we simply divide the weight of the man by 4 and get $\frac{140}{4} = 35$ lbs. weight of the counterbalance *d*.

The lever beneath, resting on the fulcrum *g*, represents the leverage of the two pulleys, *g f* being equal to *c a* and *g e* to *c b*, the weight *f* equal to the man and seat, and the weight *e* equal to the counterbalance *d*.

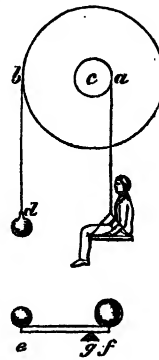


Fig. 15.

The strain upon the rope is in this case also simply as the weight suspended to it, but the load upon the pulley is only $140 + 35 = 175$ lbs.

82. In the case of Fig. 14, were motion given to the bodies, they would both move at the same rate, because the arms *e a* and *e b* of the lever on which they move are equal; but the leverage in Fig. 15 being as 4 for *g e* to 1 for *g f* for every foot *f* is made to move, *e* will move 4 feet, and yet the number of units of work done in moving *e* is just the same as in moving *f*, because 35 lbs. raised 4 feet, is equal to $35 \times 4 = 140$ units, and 140 lbs. raised 1 foot is also 140 units.

83. Fig. 16 shows a man in a seat *d*, attached to one end of a rope which passes over the pulley *c b*.

We will suppose he is seeking to balance his own weight by pulling on the free part of the rope *c e*.

The weight of himself and the seat is 140 lbs; when he allows the rope e to slip freely through his hands, there will be a pressure at b of 140 lbs.

To get a balance of pressure at c and b, he requires to apply with his hands at e pulling force equal to 70 lbs.: that is, he has to support half the weight of himself and the seat, by simply clinging by his hands at e, so that the effect will be the same as if he were to tie the end near e to the seat he sits on, in which case the parts $c\,e$ and $b\,d$ would be bearing equally.

Fig. 16.

Suppose his hands are unable to support more at e than, say 35 lbs., there will then be a pressure at c of 35 lbs., and at b of 105 lbs.; so that the seat, with its weight, would descend with the same force as if it were a weight of 70 lbs. allowed to run down freely (we here neglect the inertia of the balanced load, paragraph 78); because, assuming that the 35 lbs. friction of the hands at e be maintained during the descent, this amount of friction can balance only 35 of the 105 lbs. at b; so that 105, minus 35, equals 70, the descent of which there is no friction or force to oppose.

The action of the forces here as regards motion is simply as shown by the weighted lever $f\,g$, with the fulcrum h midway; and, as regards power, the counterbalance f for the last-named case will be as 35 to 105 for g.

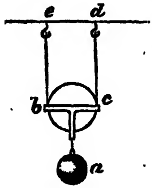


Fig. 17.

When the weight is balanced equally between the two sides c and b of the pulley, f and g will be each 70 lbs.; and when the hands are letting the rope $c\,e$ run free, f will be as 0 to 140 for g.

84. In Fig. 17, we have a pulley slung by means of a cord $b\,c$ passing round underneath it, and secured to the two hooks d and e overhead.

The opposite ends of the cord are each bearing one-half the weight of the pulley and the load *a*.

85. Fig. 18 shows one end only of the cord hooked to the roof; the other end passes over a second pulley, which takes the place of the second hook, and works on the lower end of a fixed arm, *e*.

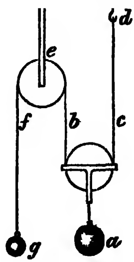


Fig. 18.

Let the weight of the slung pulley and its load *a* be 140 lbs. To balance this, there is required only a load of 70 lbs. at *g*, because the lines *b* and *c* are halving the load *a* between them, so that they are each bearing 70 lbs.; and the leverage of the upper pulley *e* being balanced, the line *f* requires weight simply equal to that which is sustained by *b*.

But, while the hook *d* has to support no more than 70 lbs., the upper pulley *e* has a load of 140 lbs.

The weight *g* is only one-half the weight of *a*; but for every foot that *a* is raised, *g* has to descend two feet, because the pulley *a*, in rising 1 foot, shortens each of the lines *c d* and *b e* 1 foot, or two feet shared between them; consequently, as *c d* and *b e* can be shortened only by this much of their united length being drawn over the pulley *e*, the weight *g* must descend at a rate to correspond.

There is no motion given to the line between *c* and *d*; the pulley *a* running up or down upon this line as it would upon a bar; but the lines *b e* and *f g* move at equal rate; that is, 2 feet for every single foot that the load *a* rises or falls; because *b e*, in addition to its own 1 foot, receives for transmission to the line *f g*, the 1 foot from the line *d c*.

The work done respectively by *g* and *a* in motion is,—for *a*, 140 lbs. $\times$ 1 foot $=$ 140 units; and for *g*, 70 lbs. $\times$ 2 feet $=$ 140 units.

86. We must bear in mind, however, that the weights we have employed in these figures effect no more than a balance of power.

When motion is wanted, additional weight must be given to the side that is required to descend, and by the amount of this addition will the speed of the motion be ruled, because any addition to the weight is equivalent simply to an increase of power.

In the first place, as much must be added as will overcome the friction of the pulley-spindles in their bearings, and a little more to balance the resistance that the rope opposes to bending round the pulley.

87. As we have already explained, (Fig. 13), the smaller the diameter of the spindle or axle, for a given diameter of pulley, the smaller the force required in the cord on the wheel-rim to overcome the friction on the spindle.

88. A very small addition may be sufficient to give motion, if the pulleys be not very small, with big axles; so that, this being understood, it is sufficiently near the mark to employ the balancing weights only.

When the power is sensibly in excess of the resistance, we get accelerated speed; that is, speed which is becoming faster and faster, as in the case of falling bodies.

It is clear that the speed must thus increase progressively, because, as in the case of the 70 lbs. unbalanced weight of the man assumed in reference to Fig. 16, paragraph 83, there is nothing to counteract the excess of power we here speak of; so that it will act with increasing momentum, as if it proceeded from a body falling through space; but, as observed in paragraph 78, the velocity will be as much less than the natural rate of motion for bodies falling freely through space, as the weight of the whole power and resistance in motion is greater than the excess weight of the power.

The force of gravity in the resistance—that is, in the

rising weight—is neutralised by the equal force of that portion of the power which balances it in the descending weight; so that the force of gravity available for motion can belong only to the excess of weight in the descending side; that is, to the excess of power.

And, as the inertia of the whole weight has to be overcome by the force of gravity acting in this small excess weight, it follows that this small portion of the whole has as many times more work to do than it would have were it detached and free to fall away by itself, as its weight is contained in the weight of the whole.

The inertia of the load opposes resistance to motion only while the motion is starting or increasing.

When the rate of motion becomes uniform, the force accumulated in the body remains within it, balancing the resistance of the inertia which it has overcome; so that, as observed in paragraph 39, the resistance of friction is all that remains for the power to deal with.

Supposing that 1 lb. of additional weight to the side *d* of Fig. 14 would just balance the spindle friction, but that we were to increase the weight of *d* by 2 lbs., so as to give motion, we could only count on those 2 lbs. as doing work, because the work done by the raising of the 140 lbs. balancing-load on the side *c* is neutralised by the lowering of the 140 lbs. on the side *d*; but the accelerated speed would be due to 1 lb. only, because the other pound weight is expended solely in overcoming friction, as explained in reference to Fig. 13.

A weight in the act of descending freely, is power in motion; but, practically, it does no work until it meets with resistance at the end of its descent. The force, however, generated in accumulative manner by the uniformly-accelerated motion may be termed work in latent form, to be expressed in sensible form on meeting with resistance.

In estimating the work done in raising weights, it is

usual to multiply the weight raised by its speed; because, while it may be easy to ascertain the weight raised, it is often difficult to weigh the power.

Were the power weighed, however, and multiplied by its uniform speed, it would be found very slightly more than equal in work units to the load lifted, added to the friction, the slight excess being employed merely to give motion.

89. Fig. 19 represents a double set of pulleys, such as are employed in tackle-blocks.

We spread them out to make the explanation simpler than if each set was shown in close order in the block. This arrangement, however, obliges us to show the fixed end of the rope attached to a distinct hook *j*, instead of to the upper block; but this makes no difference as regards the power required at *k*; though, were we seeking to know the stress on the hook *i*, it would reduce it by one-sixth.

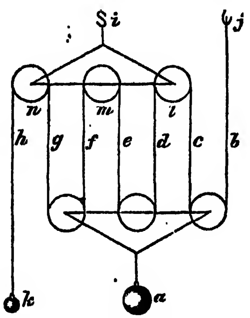


Fig. 19.

90. Let the load *a* be 140 lbs., inclusive of the weight of the lower block and its three pulleys.

We require to find the weight of *k* to balance this load, and will leave friction for the moment out of consideration.

As we found in the case of Figs. 17 and 18, that the rope which supports the slung pulley *a* has the load shared equally by the two lines *b* and *c*, we here find that it cannot be otherwise than shared equally by the six lines *b*, *c*, *d*, *e*, *f*, and *g*; so that $\frac{140}{6} = 23{\cdot}33$ lbs. is the share of the whole load to each line; and, as the

leverage of the upper pulley that supports the line *g* and *h* is balanced, it follows that *g* will be balanced by a tension equal to its own in the line *h*; consequently, 23·33 lbs. weight at *k* will balance 140 lbs. at *a*.

91. When the lower pulleys with the load *a* rise 1 foot, each of those pulleys, as before explained in connection with Fig. 18, paragraph 85, shortens each of the side lines it is slung by to the extent of the rise 1 foot, which is equal to 2 feet of rope per foot of rise; and as there are 3 pulleys $3 \times 2 = 6$ feet which the weight *k* will descend in raising the load *a* 1 foot, so that, as the power required at *k* is equal to $\frac{1}{6}$th of the load, the speed must be increased 6 times to make the work done by *k* equal to the work required by *a*: thus, $23{\cdot}33 \times 6 = 140$ units of work done by *k*, equal to $140 \times 1 = 140$ units required by *a*.

The upper pulley *l* revolves at the rate of 2 feet in the time occupied in raising *a* 1 foot; whereas the pulley *m* revolves at the rate of 4 feet in the same time, and the pulley *n* 6 feet, which is the same rate as found in the line *h k*.

This acceleration of speed in the successive stages is owing simply to the first pulley adding the rope it passes to the similar length taken up in the rise of 1 foot by the pulley next in the series.

92. In tackle-blocks, the pulleys being, for convenient handling, small in diameter, and the spindles, for strength, comparatively big, the ratio of the leverage between pulley and spindle is low, so that, to overcome the friction on the spindles produced by the weight of all the pulleys, plus the load *a*, and the power *k*, more power is required here at *k* than when the ratio of pulley to spindle is higher, as in the case of Fig. 18.

Besides, as the rope in Fig. 19 has to bend round 6 small pulleys, the power has to be additionally increased to overcome the resistance opposed to sharp bending.

93. When the rope is stiff and strong, the loss by bending is considerable, and is not a constant quantity for a given rope on a given diameter of pulley, but varies with the tension.

In Section 13 will be found a Rule for estimating the bending resistance of hemp ropes.

The means employed for the determination of the Rule differ from the simple case of suspension of Fig. 14, the cords in the experiments taking one complete turn round the pulley in place of one half-turn; but, as the resistance in the second half-turn is only a duplicate of the resistance in the first half, and as the full value of the resistance on a given diameter is determined by a bend from the horizontal to the vertical direction on an arc of 90°, the bend in the supplementary arc of 90° (completing the arc of 180°) acts as a mere counterpoise to the first, so that we employ the Rule of paragraph 203 to determine the resistance in the case of Fig. 19, and multiply by 6, the number of bends in that figure. See paragraph 219.

In ordinary practice, the power k would have to be increased by at least one-half, to cover these losses.

94. In Fig. 20 we have the Chinese windlass, formed of a cylinder a b, of unequal diameter, keyed on to a spindle e f, worked by a crank-handle g, and supporting the load by means of a pulley slung in the bight or bend of the rope, which is shown to lead from the near face of the part b, to pass round underneath the pulley, and to lead on the back face of the part a.

By means of it a man-power at the handle g may lift great weights, but, in proportion to the greatness of the load raised, is the motion given to the load small; because the power is derived, firstly, from the difference in diameter between the parts of the cylinder a and b; secondly, as regards axle resistance, from the difference between these diameters and the diameter of the spindle; and thirdly, as regards hand-power, from the difference between the mean

diameter of a and b, and the diameter of the circle $g\ h$ made by the crank-handle g in motion.

We will assume these diameters to be as follows:—
$e\,f = 1$ inch, $a = 6$ inches, centre to centre of rope, $b = 8$ inches, $g\ h = 30$ inches; or, as we will require to find the units of work done, we will give these dimensions in decimal parts of a foot. Thus, $e\,f = \cdot083$ foot, $a = \cdot5$ foot, $b = \cdot66$ foot, and $g = 2\cdot5$ feet.

In raising the load, the rope is wound off the smaller diameter a, on to the greater diameter b, by the motion given to the handle g. The circumference of a is 18·84

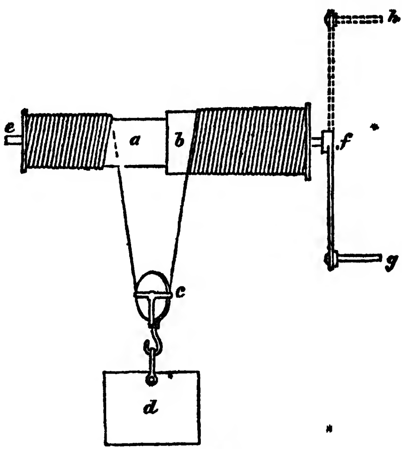


Fig. 20.

inches, and of b 25·13 inches; consequently, in every revolution, b requires 6·29 inches more rope than a is turning off, and for the reason given concerning Fig. 18, paragraph 85, the rise of the load can be only one-half this length, or 3·145 inches, which is equal to $\frac{3\cdot145}{12} =$ ·262 foot.

Let the load d be 140 lbs. weight, including the weight

of the pulley c. Then $140 \times \cdot262 = 36\cdot68$ units of work done per revolution of the cylinder, and as the handle g by which the power is applied, moves round with the cylinder, we find the circumference in feet of the circle $g\,h$; then find the number of times the rise of the load is contained in this, and divide the load by this number. The result is the weight of the power required at g, when the spindle friction is disregarded.

The circumference of $g\,h$ is $2\cdot5 \times 3\cdot1416 = 7\cdot85$ feet, so that $\frac{7\cdot85}{\cdot262} = 30$ times the rise of the load is contained in the circle $g\,h$; and $\frac{140}{30} = 4\cdot66$ lbs. power required at g to balance the load d.

95. The load d, 140 lbs., being here merely balanced by the 4·66 lbs. in the circle $g\,h$, it follows that (neglecting friction) as a very small addition to the weight of the power at $g\,h$ serves to overcome the inertia of the load d, plus the weight $g\,h$, the inertia is equally overcome when the small addition is transferred to the load d, and, as the load can perform no more work in descending ·262 foot than is required to raise it that distance, the power expended in giving motion is the same, whether we have the resistance at d or at $g\,h$; because, the larger circle $g\,h$ being completed in the same time as the smaller circles of $b\,a$, we have what is wanting in weight at $g\,h$, compensated by the greater velocity; so that, when we multiply the circumference of $g\,h$ by the 4·66 lbs. resistance, we get the same as found for d, thus,

7·85 feet $\times$ 4·66 lbs. $= 36\cdot68$ units of work performed in the circle $g\,h$.

We before observed that it does not matter in what direction the power is acting; that is, whether vertically or horizontally; or, whether it be exerted in pure lifting, as in Fig. 11, or in overcoming combined sliding and rolling friction, as in Fig. 10.

The smaller power, therefore, at *g*, moving in a circular path, is of equal effect to the greater power *d* descending vertically, excluding friction. (Paragraph 97.)

When the pressure exerted on *g* is the motive power, it simply overcomes the resistance opposed to it by the load *d*.

When the pressure is applied by the hand, that hand moves in the circular path, but we would get the same effect by hanging a weight equal to that pressure, over a pulley of the same diameter as the circle that the handle *g* makes, in which case, this weight, acting as the power, would descend vertically, while the load *d* would rise vertically.

96. There are some points of difference, however, between the hand and the descending weight, which we will here mention. They concern the pressure on the axle.

In estimating the friction, we must find for the weight of the load *d*, and pulley *c*, the cylinder *a b* with spindle and crank-handle, and exclude the weight of the hand pressure.

We include the whole weight of the power *k*, in the case of Fig. 19, because the pull is constantly downward; but here, in Fig. 20, the power at work upon the crank may be considered as pressing downward during half the revolution, and drawing upward during the other half. When drawing upward, it is of course lightening the pressure of the spindle upon its bearings, so that the downward pressure is balanced or neutralised in the course of one revolution, by the upward relief.

Let us call the total load 200 lbs., we have already stated that the friction of motion on wheel axles is equal to ·07 of the weight. This is termed the "coefficient" of axle friction. We shall employ it frequently.

200 lbs. × ·07 = 14 lbs. frictional resistance to be overcome.

The diameter of the circle the power moves in is 80

inches, and of the spindle 1 inch, consequently the leverage is 80 to 1 in favour of the power, so that we divide the frictional resistance by 80, and get $\frac{14}{80} = \cdot46$ lb. at *g*, the required force to overcome 14 lbs. on the circumference of the spindle *e f*. We therefore add this to the balancing power, $4\cdot66 + \cdot46 = 5\cdot12$ lbs., required at *g* to balance the load *d* and the spindle friction.

Now, if we leave out of account the resistance of the rope to bending, any small addition to 5·12 lbs. power at *g* will give rising motion to the load *d*.

When the load *d* is made to act as the power, by slowly falling, the friction has to be overcome by a greater force at the end of a shorter lever. Thus, the diameter of the cylinder at *a* is only as 6 to 1 for the spindle, and the diameter at *b* as 8 to 1.

When the load descends, the rope on the diameter *b* is running off, and on the diameter *a* it is winding on, but at rates differing according to the respective circumferences.

97. The diameters *b* and *a* share the burden of the load *d* equally between them, but the half borne by *b* is at the end of a lever represented by the radius, 4 inches long, whereas *a* bears its half at the end of a 3-inch lever.

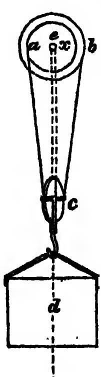


Fig. 21.

When looking endwise at the cylinder, the centre of the load *d* is found not in vertical line with the centre of the cylinder, but to one side, as shown in Fig. 21 by the line *c x*, which is midway between *a* and *b*, so that the leverage distances *x b* and *x a* are equal; and the distance *a b* being only 7 inches, the leverage distance *x b* is only $3\frac{1}{2}$ inches.

The load *d* is balanced on the line *x c*, but the leverage for axle friction has relation to the centre line *e d*, as 4 on the side *b* to 3 on the side *a*.

Were a and b each of four inches diameter, $14 \div 8 =$ 1·75 lbs. on rim would balance axle resistance, but being as 3 to 4, with the load d halved between them, and with the work done by d equal to half the difference between the 3 and 4 rate, we have $\frac{70 \times 3 \cdot 5}{3} = 81 \cdot 66 - 70 =$ 11·66 lbs. free power at b.

Assuming all the parts of the 200 lbs. weight on axle to move at the same rate, we have $200 \div 11 \cdot 66 = 17 \cdot 1$ times the downward motion of d is less than the motion of weight falling freely. (Paragraph 88.)

SECTION VII.

98. In Fig. 15 we have a weight of 35 lbs. on a diameter of 4 feet balancing 140 lbs. on a diameter of 1 foot. We will now see how much we can reduce the 4 feet diameter when we apply the 35 lbs. power by

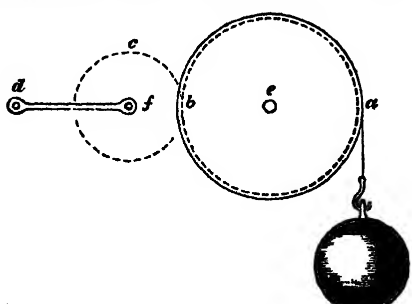


Fig. 22.

means of a crank handle and small toothed pinion working into a toothed wheel which takes the place of the 4 feet pulley.

Let the load pulley a, Fig. 22, be 1 foot diameter, as

in Fig. 15; let the pinion c be 7 inches diameter, and the length of the crank $f\,d$ be 15 inches.

The diameter of the wheel *b* we require to find. The radius of *c* being 3·5 inches, and the crank 15 inches, we have the leverage here in the ratio of 4·286 to 1. Consequently, a power of 35 lbs. at *d* will balance a resistance of $35 \times 4{\cdot}286 = 150$ lbs. on the rim of *c*, and as this is greater than the load, we divide the load by it to get the diameter of the wheel *b*, thus: $140 \div 150 = {\cdot}933$ foot for *b*, when the diameter of *a* is 1.

99. Fig. 28 shows the action of the power operating by leverage. It shows also that the strain upon the teeth

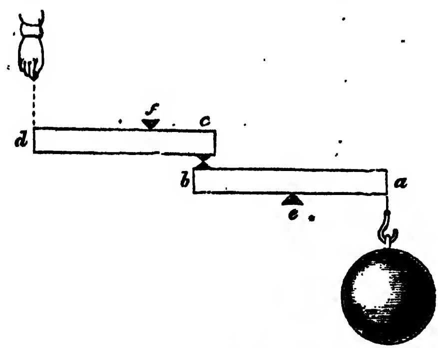


Fig. 28.

of the pinion must be equal to the strain upon the teeth of the wheel.

When we come to deal with axle friction, however, where two wheels are in gear, we will find that though the strain upon the teeth be equal, and the full strain be transmitted through the receiving teeth, the axles of *b* and *c* are not enduring equal pressure: thus, when the fulcra *e* and *f* of Fig. 28 represent the axles of the wheels *b* and *c*, we can readily see that, leaving the weight of the wheels or levers out of account, the fulcrum *e* has to

bear a pressure of $140 + 150 = 290$ lbs., and the fulcrum f a pressure of only $150 + 35 = 185$ lbs.

100. By reducing the diameter of the pinion *c*, and thereby giving the crank a higher ratio of leverage, the diameter of the wheel *b* may be still further reduced below the diameter of the load pulley *a*; but this reduction would bring more strain upon the teeth, because of the reduced leverage of the wheel *b* in relation to the greater leverage of the load pulley *a*.

Supposing the crank *d* were equal only to the length of the radius of *c*, the power has no more advantage than if it were applied directly to the rim of the wheel *b*, because, being equal to the radius of the pinion, and the teeth of the pinion and the wheel being equally strained, it would encounter no more resistance in the circle of the teeth of *b* than it does in the circle of the teeth of *c*.

101. In Fig. 24 we have the man-load weighing

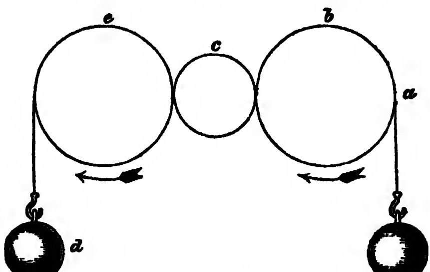


Fig. 24.

140 lbs.; the load pulley *a* and the toothed wheel *b* together on the same axle, are here each 12 inches, and the pinion *c* is 7 inches diameter, as in Fig. 22. But, in order to explain the use of a single intermediate pinion, we put a second toothed wheel *e* into gear with *c*, and suspend from its rim the weight *d* that is to act as the power.

We require to find the weight of *d*.

Now, as we have already shown that the teeth in the wheel and pinion *b* and *c* are equally strained, it is clear that the teeth of the wheel *e*, no matter what its diameter may be, can be bearing no more than that strain, and, as that strain here represents the load at *a*, it follows that the counterbalancing weight suspended from the equally-strained rim *e*, must be simply equal to the load, so that the power *d* weighs 140 lbs., and the pinion *c* is here serving merely to communicate motion from *b* to *e*, and make the direction of that motion the same in the two wheels, as indicated by the arrows.

Were the pinion removed, and the wheels *e* and *b* brought into close gear, the motion given to *e* would be in the opposite direction to that of *b*, and the counterbalancing weight *d* would in that case require to be suspended from the opposite side of *e*, that is, from the side nearest to *b*, but the weight of *d* would require to be 140 lbs. as before; so that no advantage at all is gained as regards power by the use of the wheel and intermediate pinion *e* and *c*, because the weight *d* would serve the same purpose if suspended simply as in Fig. 14.

102. We will now ascertain the advantage when the power is acting at the end of a train of toothed wheels and pinions, as in Fig. 25.

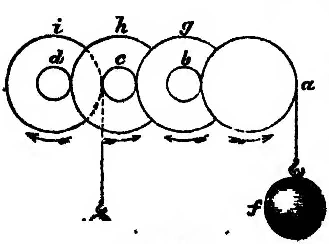


Fig. 25.

We have shown by Fig. 24 that a single intermediate pinion merely transmits power, in the same degree as it receives it, minus the friction due to its own weight and the pressure, so that the circumference of *c* is enduring the same strain as the circumference of *b* in gear with it.

In Fig. 25, however, when the pinion *b* receives the

strain at the end of a short lever represented by the radius, and transmits it through the longer lever represented by the radius of the wheel g to the next pinion c, and so on to the power e, it is clear that as the greater leverage of g enables a smaller weight to balance the load acting on the rim b, a still smaller will suffice at h, and that a weight less than required at h will suffice at i.

Let the wheels a, g, h, and i be each 12 inches in diameter, and the pinions b, c, and d be each 4 inches in diameter, and the load f be 140 lbs. We require to find the weight of e.

103. Now, as the diameters of each couple of wheel and pinion are in the ratio of 3 for the wheel to 1 for the pinion, thus $\frac{12}{4} = 3$, and as the strain on the rim of the pinion b is equal to the load f, when friction is not considered, we begin at the point b to seek for the weight e; and, as the gain in power for each pair of wheel and pinion is 3 times, we have the gain in g b repeated in h c, and again in i d, so that, supposing the strain on b to equal 27 lbs., the strain on c would equal $\frac{27}{3} = 9$ lbs., and the strain on d $\frac{9}{3} = 3$ lbs., and the strain at the rim i would equal $\frac{3}{3} = 1$ lb., which would be the weight required for e; but the load f is 140 lbs., therefore $\frac{140}{3} =$ 46·66 lbs. for c, $\frac{46{\cdot}66}{3} =$ 15·55 lbs. for d, and $\frac{15{\cdot}55}{3} =$ 5·18 lbs. for the rim i, which is the weight of e to balance 140 lbs. at f when friction is neglected.

When we wish simply to find the weight of e without reference to the strains in the intermediate gearing, we proceed thus, $3 \times 3 \times 3 = 27$ times gain in leverage, so that $\frac{140}{27} = 5{\cdot}18$ lbs. at e.

We have here been working backwards from the load to the power, as we wished to find the latter. Were we to employ *f* as the slow power, it would balance only the 5·18 lbs. quick motion load at *e*.

104. We are less liable to error in measuring the forces, when we employ the respective numbers of the teeth in the wheel and pinion of each pair, in place of using the diameters, or radii; and we get the same result by multiplying together the number of teeth in the train of pinions, and using the product as a divisor of the product of the teeth of the train of wheels multiplied together: thus,—supposing the pinions had each 20 teeth, and the wheels 60, we get $20 \times 20 \times 20 = 8000$, and $60 \times 60 \times 60 = 216000$, then $\frac{216000}{8000} = 27$ times the pressure on the rim of *i* is contained in the pressure on the rim *b*; and also, 27 times the motion of the rim *b* is contained in the motion of the rim *i*.

It is clear that, when the teeth are of the same size, the number contained in the circumference of a wheel 12 inches in diameter must be just three times as many as are contained in the circumference of a pinion 4 inches in diameter.

105. It is equally clear, however, that the teeth of *d* need not be so strong as the teeth of *b*; but, as the leverage is ruled by the diameter, the only advantage that would be got by making the teeth finer would consist of less axle friction, owing to the greater lightness of the wheels.

106. In proportion to the power gained as we advance from the load pulley *a*, to the power weight *e*, is the speed increased, because the rim *g* moves at 3 times the rate of the rim *b*; consequently *c* is making 3 revolutions for every one of *b*, and *d*, at the end of the train, is making $3 \times 3 = 9$ revolutions for 1 of *b*, and, as the rim *i* is moving at 3 times the rate of the rim *d*, we have the power *e* on the rim *i*, moving $3 \times 3 \times 3 = 27$ feet for every 1 foot of the rim of the pinion *b*, and therefore of the load *f*,

which is the same result as got when we employed the numbers of the teeth to find it.

107. In estimating the friction upon the axles, we employ the dead weight of the gearing, the load *f*, and the power *e*, and add the pressure on the intermediate wheels, because that pressure acts in the manner of a burden upon the supporting axles. When acting upwards, however, against the dead weight of the wheel or lever, the axle pressure is relieved to an extent equal to the upward force, but of this we will speak presently.

108. Fig. 26 shows the first wheel and pinion of Fig. 25, enlarged for greater clearness.

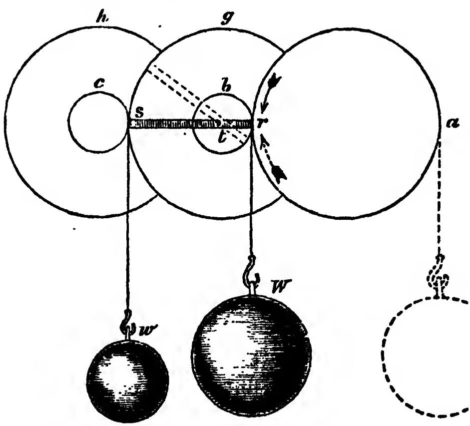


Fig. 26.

The lever arm *t r* represents the leverage of the pinion *b*, and *t s* the leverage of the wheel *g*. As before explained, the strain at *r* is equal to the load *f*, so that the load pressure at *r* is 140 lbs., represented by the hanging weight W; and, as we have already found that the pressure at *s* is equal to 46·66 lbs., we represent the power required to balance that, by the smaller hanging weight *w*, and simi-

larly on to the end of the train, where we have 5·18 lbs. at *e*.

We will suppose the wheel and pinion *g* and *b*, with their axle, to weigh 20 lbs.

We shall presently see that this weight is borne by the wheels in gear with *g b*, so that for this particular couple, the axle pressure at *t* is equal only to the pressure on the teeth, thus, 140 + 46·66 = 186·66 lbs., which will appear in our estimate when finding the total quantities for the whole train.

109. The crossed arrows in Fig. 27 show the direction in which the power acts in lifting the load *f*. The dotted

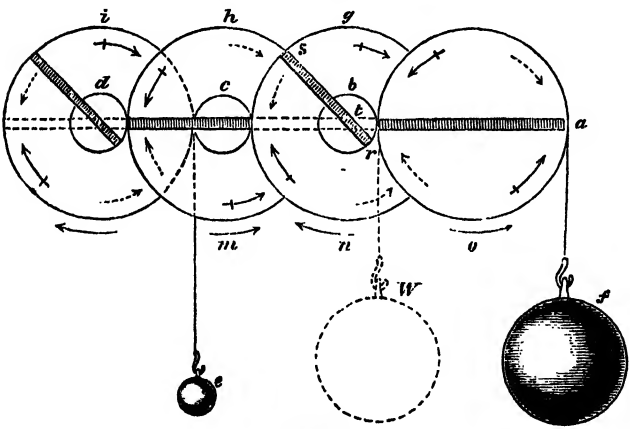


Fig. 27.

arrows the direction of the resistance due to the load. The plain arrows *m n o*, the direction of motion when the load *f* is being raised.

Now, it matters not as regards axle friction, whether we use the pressure of the load resistance indicated by the dotted arrows or the power force indicated by the crossed arrows, because in all these estimates we are employing

power that just suffices to overcome the resistance. As we before explained, any power in excess of this would give constantly increasing or accelerated motion, which is not wanted.

Neither does it matter as regards the total friction in the entire train of wheels whether the power at the ends of the levers be exerted in lifting or in pressing downward, because, though in downward pressure, to give motion to the wheel next in gear, the axle is relieved of a certain portion or of the whole weight of the wheels which are keyed upon it, this relief is at the expense of the axles next to and in gear with it, because, as the weight must be borne by something, it is supported by the teeth of the wheels on those next axles in the manner represented by the levers in Fig. 28.

110. In Fig. 28 we show the 3 to 1 leverage of the wheels of Fig. 25.

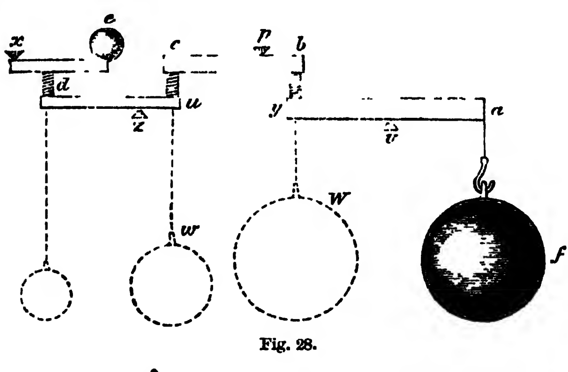


Fig. 28.

The load *f* is 140 lbs., the power *e* is 5·185 lbs., and the intermediate pressures on the teeth are, for *b y*, 140 lbs., *c u*, 46·66 lbs., and *d*, 15·55 lbs.

111. We represent the intermediate teeth pressures by spiral springs, which exert equal force on the upper and lower lever ends, so that the axle of the wheel that is giving

is subjected to the same pressure as the axle of the receiving wheel.

112. We will now find the total friction in the entire train, and will give the necessary explanation afterwards. The levers *a y*, *b c*, *u d*, and *e x*, which are each taking the place of a wheel and pinion, we will assume to weigh 20 lbs. each, so that $20 \times 4 = 80$ lbs. weight for the 4 sets of levers, or of wheel and pinion.

$$\begin{aligned} a + y &= 140 \quad + 140 &&= 280{\cdot}0 \\ b + c &= 140 \quad + \ \ 46{\cdot}66 &&= 186{\cdot}66 \\ u + d &= \ \ 46{\cdot}66 + \ \ 15{\cdot}55 &&= \ \ 62{\cdot}21 \\ e &= \ \ \ 5{\cdot}18 &&= \ \ \ \ 5{\cdot}18 \end{aligned}$$

534·05 lbs. pressure.

To this we now add the 80 lbs. weight of wheels and pinions represented by the levers, and get 614 lbs. total pressure on the axles. From this, however, we must deduct the upward force of the pressure *d* from the combined weight of *x e* + *c*, because the difference alone is operating in friction upon the axle *x*, as we shall presently explain; so that $614{\cdot}05 - 15{\cdot}55 = 598{\cdot}50$ lbs.

We now multiply this by the coefficient of friction for axles, viz., ·07, and get the total frictional resistance acting on the circumference of the axles. Thus, $598{\cdot}50 \times {\cdot}07 = 41{\cdot}89$ lbs.

113. To overcome this we have to make an addition to the 5·18 lbs. weight which is acting as the power at *e*, and to find how much has to be added, we may assume, as the wheels and pinions are respectively of uniform diameter, that the friction is all borne by one of the axles, that is, by the axle of *d i*, Fig. 25, on the outer rim of which the power is applied.

The strain on this wheel being comparatively light, we may make the axle proportionately small, say ¾ inch diameter, or ·75 inch expressed in decimals.

Now, as the wheel *i* is 12 inches diameter, we have its leverage in the ratio of 16 to 1 for the axle, $\frac{12 \cdot 00}{\cdot 75} = 16$. We therefore divide 41·89 by 16, and get 2·618 lbs. required to overcome the frictional resistance on the axles, so that 5·185 + 2·618 = 7·803 lbs. required at *e* to place all the forces in that condition of balance that any small additional weight at *e* would produce motion.

We have here treated the intermediate pairs of wheel and pinion as if on axles of the same size as that of *i*; that is, of ¾ inch diameter.

Were we to make them very different in diameter, we would have to find the ratio of leverage, and treat the friction of each axle separately.

For a given diameter of wheel, the nearer the diameter of the axle approaches the dimensions of a point, the less is the resistance of the friction opposed to the power in motion.

114. We will now find the weight which presses upon each axle separately.

The pressures exerted by the wheel teeth are transmitted entire to the axles, because the receiving teeth contain the resistance which in our estimate just balances the power in the driving teeth; and the stiffness of the wheels makes the pressure on the axles equal to the pressure between the teeth in contact.

The fulcra points *v*, *p*, *z*, and *x*, of Fig. 28, represent the axle centres of Fig. 25.

The weight of the lever *a v y* is wholly borne by the fulcrum *v*.

The arm *p b* of the lever *p b c*, being as 1 only, to 3 for *p c*, three-fourths, equal 15 lbs. of the simple lever weight is borne by *y*, and therefore is added to the pressure on the fulcrum *v*, while one-fourth, equal 5 lbs., is borne by *u*, to be added to the other pressures upon the fulcrum *z*.

115. The centre of gravity of the wheel and pinion is

in the centre of the axle, so that as the fulcra points *v*, *p*, *z*, and *x*, represent the axle centres, we must regard the weight of the levers as centred or balanced on these points.

Hence, when the distance *p b* is 2 inches, and the distance *p c* is 6 inches, we have $\frac{6}{2} = 3$ times as much of the weight of the wheel and pinion borne by the support at *b* as we have at *c*.

The only pressure upon the fulcrum *p* is upward from the resistance forces at *b* and *c*. The point of greatest pressure lies on the upper or the lower side of the axle circumference, according as we have shown the fulcra points up or down, with the exception of *x*, as we will presently explain.

The fulcrum *z* bears the pressure *c*, the whole weight of the lever *u d*, the pressure *d*, and one-fourth part of the weight of the lever *b p c* at *c*.

Further, the weight *e* added to the weight of the lever *e x*, less the difference between them and the pressure *d*; thus, $5{\cdot}185 + 20 = 25{\cdot}185 - 15{\cdot}55 = 9{\cdot}63$ lbs. remain as pressure upon the axle *x*.

The 15·55 lbs. downward pressure here subtracted being balanced by the upward force at the end *d* of the lever *u d* appears only once when we put the whole into arithmetical form.

On looking at the figure, it may appear as if the weight *e* ought to be included in the weight borne by the fulcrum *z*, but it is right to treat it as if forming part of the weight of the lever *e x*; because the 15·55 lbs. pressure at *d* represents its leverage effect.

Were we to add it to *d*, we would thereby be simply adding effect and cause together, and making them act, when thus joined, as effect greater than there was cause for.

116. We may be able to show this more clearly by means of Fig. 29, in which the lever *e x* is in action by

itself, and producing by the 5·185 lbs. weight at e, a pressure of 15·55 lbs. at the point d, represented by the spiral spring underneath.

$x\,e$ = the radius of the wheel, is 6 inches,
and $x\,d$ = the radius of the pinion, is 2 inches.

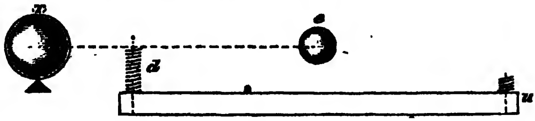


Fig. 29.

Let us assume the point of resistance d in the pinion teeth to be the fulcrum on which the lever $x\,e$ may rock, with the whole 20 lbs. weight of the wheel and pinion in effect gathered in condensed form round the centre of gravity in the axle centre x, and the 5·186 lbs. power-weight in similar condensed form at the other end of the lever.

Now, $d\,x = 2$ inches, and $d\,e = 4$ inches; so that the ratio of leverage is $\frac{4}{2} = 2$ for $d\,e$ to 1 for $d\,x$.

We have to ascertain how much of the weight of x the weight e can balance; the unbalanced quantity will be the weight left for the axle carriage at x to bear; thus, $x \times 1 - e \times 2 = 20 \times 1 - 5{\cdot}186 \times 2 = 9{\cdot}63$ lbs. unbalanced.

And further, as regards the effect of this upon the fulcrum d, which can sustain no more than a pressure of 15·55 lbs., we make x the fulcrum, seeing that its gravity is more than the weight e can overcome; and, as the leverage $x\,e$ is 3 times as much as the leverage $x\,d$, we have the pressure at d equal to $5{\cdot}185 \times 3 = 15{\cdot}55$ lbs.

117. The weight of an actual lever such as $e\,x$ in Fig. 28 would operate in addition to the power e, so as to increase the pressure at d; but we have here employed the lever merely to represent the radial lever-

age of the wheel and pinion; and, as their weight is balanced on the axle which corresponds with the fulcrum x, so that the centre of gravity is the centre of the axle, and therefore the fulcrum-point, we have no centre of gravity to find between e and x, as we would have in the case of an actual lever; and, as the weight at x of the lever, or wheel and pinion $e\,x$, and the power e, are the only forces acting downward, against the only force d, acting upward—that is, $20 + 5{\cdot}185 = 25{\cdot}185$ —we simply find the difference as before, $25{\cdot}185 - 15{\cdot}55 = 9{\cdot}63$ lbs. downward pressure left unbalanced for the axle represented by the fulcrum x to support.

In Fig. 28 we have shown the fulcrum x on the top side of the lever, whereas the unbalanced pressure is here found acting downward.

Were the weight of the wheel and pinion, say only 5 lbs., so as to make, when added to e, $5 + 5{\cdot}185 = 10{\cdot}185$, the fulcrum as placed in Fig. 28 would be right, because there would then be $15{\cdot}55 - 10{\cdot}185 = 5{\cdot}365$ lbs. unbalanced pressure acting upward.

118. But, were the weight of the wheel and pinion equal to $10{\cdot}365$ lbs., so as to make, when added to e, $10{\cdot}365 + 5{\cdot}185 = 15{\cdot}55$, the force acting upward at d would be exactly equal to the combined forces acting downward, and there would be no pressure either upward or downward at x, where the weight of the wheel and pinion is centred.

This difference between the action of the forces in the wheel and pinion represented by $e\,x$, and the action in the other levers of the series, is caused by the power e being applied on the same side of the axle as the resistance d.

The lever $e\,x$ is shown inverted, because the direction of motion when the 140 lbs. load is being raised, requires that the power be applied on the side of the wheel i, which is nearest the load f, as shown in Fig. 25.

Were there four pair in place of only three pair of wheel and pinion, there would be no need to invert the first lever as *e x* is, and the question of leverage and of burden on the axles would be simplified.

119. We will now put into arithmetical form all these quantities that we have found for the axles separately:—

$$a+y+ay+pb = 140 + 140 + 20 + 15 = 315$$
$$b+c \ldots\ldots\ldots\ldots = 140 + 46{\cdot}66 \ldots\ldots\ldots\ldots = 186{\cdot}66$$
$$u+d+ud+pc = 46{\cdot}66 + 15{\cdot}55 + 20 + 5{\cdot}0 = 87{\cdot}21$$
$$x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = 9{\cdot}63$$
$$598{\cdot}50$$

lbs. total pressure upon the axles, same as before found for the whole train when treated generally. (Paragraph 112.)

120. Now, as the work expended in raising the load *f*, Fig. 25, is simply 140 units for every foot of rise, when the friction of the axles is left out of account, we find that this is just equal to the work performed by the power *e*, when we multiply the weight of that power by its greater speed: thus, 5·185 × 27 times f- speed, equals 140 units.

We may express this in different ways without altering the value, though we would affect its direct application to the actual dimensions of the power and speed employed, because the different terms would represent equivalents only.

Thus, we may say, 1 lb. pressure moved 140 feet, or 5·185 lbs. pressure moved 27 feet, by the rim of the wheel *i*, in the same time as the load *f* occupies in rising 1 foot.

The latter expression, to lie parallel with the former, simply requires us to suppose each lb. of the 5·185 lbs. moved separately to the extent of 27 feet, so as to make the total equal to 1 lb. raised to 5·185 times 27 feet, which is 140 feet.

We have already explained, in reference to Fig. 25, how this 27 for speed is got. The leverage in Fig. 28

shows the reason in plain form; as it is clear that, when the two arms of a lever—say *p b* and *p c*, of that figure, are in the ratio of 1 for *p b* to 3 for *p c*, the motion at *c* must be 3 times as much as the motion at *b*, and as we have the three levers, *b c*, *u d*, and *e x*, all in the 3 to 1 ratio, we get $3 \times 3 \times 3 = 27$ for the motion of the weight *e*, for every 1 of the weight *b* or *a*.

As the fulcrum *v* is in the middle of the lever *a y*, the ends *y* and *a* move at equal rate.

When the weight *e* moves at the rate of 50 feet per minute, we find the units of work done by it thus, 5·185 lbs. × 50 feet = 259·25 units of work per minute, a we find the motion of the load *a* thus, $\frac{50}{27} = 1{\cdot}852$ fee per minute, and 140 lbs. × 1·852 = 259·25 units of work per minute, expended upon the load *a* in raising i. 1·852 feet.

121. In this, for the sake of simplicity, we have excluded the frictional resistance on the axles, as the force expended by the addition made to *e* in overcoming it does not reappear in the rising of the load *a*, but is, as it were, a loss between the power and the load.

We found, however, in connection with Fig. 28, paragraph 113, that the frictional resistance with the weights used is equal to 41·89 lbs., and that an addition of 2·618 lbs. to the 5·185 lb. power *e* is required to overcome it; so that, in estimating the work performed by the power in overcoming the gravity of the load *a*, and the frictional resistance on the axles when the motion of the power is at the rate of 50 feet per minute, we simply multiply the total weight *e* with its additions by the speed, thus, 5·185 + 2·618 = 7·803 lbs. weight of *e*, and 7·803 × 50 feet = 390·15 units of work per minute. We find the units of work expended in overcoming the friction, thus, 390·15 — 259·25 = 130·90.

122. As the ratio of friction to the load pressure is prac-

tically the same for all speeds, the 2·618 lbs. power for friction is constant for the given total weight and ratio of axle diameter; whereas, the 5·186 lbs. force merely balances the load.

SECTION VIII.

123. The wheels in the train, Fig. 25, are made to make the same number of revolutions per minute as the pinions by means of "keys," or long slips, of iron, which are imbedded in the axles to half their depth, the other half projecting above the surface of the axle, so as to be imbedded in the centre metal of the wheels and pinions, as shown in Fig. 30, where *a* is the wheel of 12 inches, *b* the pinion of 4 inches, and *c* the axle of $\frac{3}{4}$-inch diameter. We require to know the pressure upon the key.

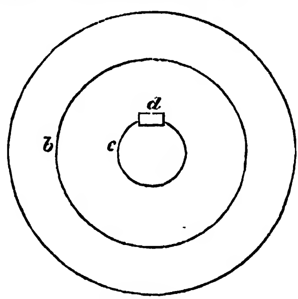


Fig. 30.

Now, as regards friction, it does not matter whether the axle be fixed and the wheel revolving round on it, or whether they be keyed together, save in this much, that when keyed together the small weight of the axle is increasing the friction.

The question of keying, therefore, is mainly of consequence when the power is communicated to one of two wheels upon an axle to be transmitted through the other to a third wheel on another axle.

124. Let the wheel and pinion *a* and *b*, Fig. 30, represent the wheel and pinion *g* and *b*, of Fig. 25.

We have, therefore, 140 lbs. pressure on the rim of *b*, and 46·66 lbs. on the rim of *a*.

Now, the ratio of leverage is $\frac{12}{\cdot 75} = 16$ for *a* to 1 for *c*, and $\frac{4}{\cdot 75} = 5{\cdot}33$ for *b* to 1 for *c*, consequently, $140 \times 5{\cdot}33 = 746$ lbs. pressure on the key at the surface of the axle, from 140 lbs. force on the rim *b* when opposed by $46{\cdot}66 \times 16 = 746$ lbs. pressure at the same point from 46·66 lbs. force at *a*. We have here to multiply the force on the rim by the ratio of leverage, because it is at the end of a lever which is longer than the lever of the resistance at the axle.

125. Supposing the axle and the pinion *b* were made of one piece, and of the same diameter, that is, 4 inches, the wheel *a* being 12 inches, and the key placed as we here show it, on the rim of *b* in Fig. 31, we have the stress upon the key equal simply to the load *d*, which is, say, 140lbs. as before, *e* being 46·66 lbs. And, as the tendency here is for the wheel and pinion to run in opposite directions, the key alone preventing them from so doing, we must regard them as acting from their common centre *x*, with the ratio of leverage 3 to 1 as in Fig. 28, so that we have $140 \times 1 = 140$ lbs. pressure on the key at the surface of the 4-inch axle from 140 lbs. load on rim of *b*. And $46{\cdot}66 \times 3 = 140$ lbs. pressure at the same point from 46·66 lbs. force on the rim of *a*.

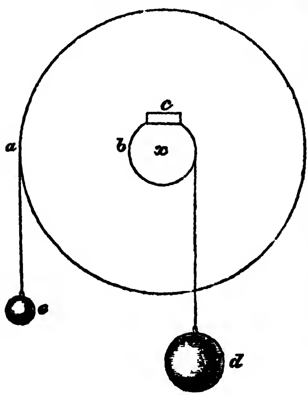


Fig. 31.

126. The power *e* in this case has less leverage advan-

tage in overcoming the frictional resistance on the axle than in the case of Fig. 28, in which the axle is so much smaller. Here we have the ratio as 1 to 3 only in place of 1 to 16.

We will find the additional weight required here by *e* to balance the friction, and for convenience will use the same weight of wheel and pinion and axle as in Fig. 28, viz. 20 lbs.

140 + 46·66 + 20 = 206·66 lbs. weight pressing downward. This multiplied by ·07 gives 14·46 lbs. friction on the surface of the axles, and 14·46 lbs. divided by 3 for leverage, gives 4·82 lbs. to be added to the power *e*, so that 46·66 + 4·82 = 51·48 lbs. required at *e* to balance the load *d* and the frictional resistance on the axle.

Any small addition either to the power *e* when thus increased, or to the load *d*, would give motion when *d* is first increased by the whole of axle friction.

127. Sometimes a machine shaft is made with the journals, or parts which work in the bearing carriages, of unequal diameter.

Now, when the shaft is supported at the ends merely, the total amount of friction is equal in the journals when the load is equal, notwithstanding the inequality of the diameter, but the power overcomes the friction with greatest ease in the case of the smaller diameter, because of the ratio of the leverage being higher.

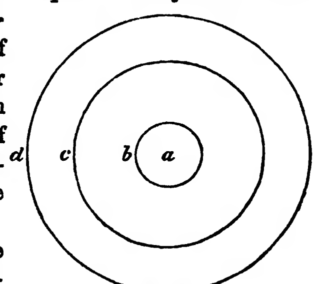


Fig. 32.

Let *a* in Fig. 32 be the centre of the shaft, the radius *a b* of the smaller journal equals, say, 3 inches, the radius *a c* of the bigger journal equals 4 inches, and the radius *a d* of the power equals 12 inches.

Let the load on each of the two journals equal 1,200 lbs., and the power at *d* equal 50 lbs. The greater radius *a d* is as 4 to 1 for *a b*, and as 3 to 1 for *a c*, so that as $1200 \times {\cdot}07 = 84$ lbs. frictional resistance on each of the journals, we have simply to divide this by the respective ratios of leverage, to find what proportion of the power is expended in overcoming this resistance in each journal; thus, $\frac{84}{4} = 21$ lbs. at *d* for *b*, and $\frac{84}{3} = 28$ lbs. at *d* for *c*; we add these two quantities together, and get $21 + 28 = 49$ lbs. power required at *d*, to balance the whole frictional resistance, which leaves only 1 lb. free to produce other work in motion.

128. Fig. 33 represents the end of a shaft *a*, turning freely upon two anti-friction rollers *b* and *c*. Let the

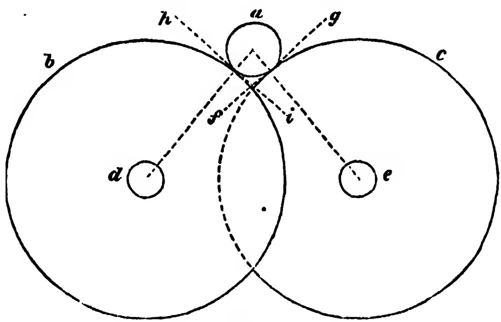


Fig. 33.

diameter of *a* be 2 inches, and the load 200 lbs., the diameter of the bearing rollers *b* and *c* 12 inches, and the weight of each 10 lbs., and the diameter of the spindles *d* and *e* 1 inch.

Were the shaft end *a* rubbing in the bushes of an ordinary carriage, the friction on its surface would be $200 \times {\cdot}07 = 14$ lbs. frictional resistance. We have now to find how much less it is when the pressure rolls upon the rims of

the bearing rollers *b* and *c*, and does not appear in rubbing form until it reaches the axles *d* and *e*.

We see that the load *a*, being borne equally by the two rollers, has its weight equally divided between them, so that $\frac{200}{2} = 100$ lbs. each, borne by *b* and *c*.

This burden is actually increased by a certain amount of thrust, owing to the wedge form of the hollow in which *a* lies, but we will disregard it as small in quantity, unless the centres *d* and *e* are so far separated as to let *a* come low down. We will, therefore, simply add the weight of the rollers to this load, and find the power which is required on the rim of the rollers to balance the friction on the axles *d* and *e* at the point of motion.

This power will be derived from *a*, and will represent the resistance opposed to its rolling motion on the rims of *b* and *c*. $100 + 10 = 110$ lbs. $\times$ 2 rollers $= 220$ lbs. $\times$ $\cdot 07 = 15\cdot 40$ lbs. friction on the axles *d* and *e*.

The rollers being 12 inches in diameter, and the axles 1 inch, the ratio of leverage is 12 for *b* and *c*, to 1 for *d* and *e*.

We employ this ratio thus, $\frac{15\cdot 40}{12} = 1\cdot 28$ lbs. frictional force required from *a* in rolling on the rims of *b* and *c*, to balance the frictional resistance due to the whole pressure on the axles *d* and *e*.

129. But, it must be borne in mind that the saving here is in driving power alone, because the tear and wear due to rubbing friction is actually greater in *d* and *e*, in the proportion of 15·40 to 14, for *a* alone, in an ordinary carriage.

This tear and wear, however, has been transferred to the end of the short arm of a lever, represented by the radius of the axles *d* and *e*, whereas the power is at the end of the long arm, represented by the radius of the rellers *b* and *c*.

The 14 lbs. friction on the circumference of *a*, constitutes the driving power in action at the end of the long arm.

The leverage is 12 to 1, so that 1 lb. at *a* can balance 12 lbs. on the circumference of *d* or *e*, and, as we have found that the whole resistance at *d* and *e* is balanced by 1·28 lbs. at *a*, it follows that the employment of the anti-friction rollers has saved to the driving power at *a* 14—1·28 = 12·72 lbs.

We shall have occasion, by-and-by, to speak of wedge-grip in V-grooved pulleys; our observations then will apply equally to the case of *a* here in the hollow between the two rollers; and the angle which the sides form will be found by drawing the lines *f g* and *h i*, Fig. 38, through the points of contact between *a* and the rollers *b* and *c*, and at right angles respectively to lines drawn from the centres of *d* and *e* to the centre of *a*.

SECTION IX.

180. When power has to be transmitted from one point to another with precision, shafting and tooth gearing must be employed; but, when particular precision is not required, belts are often used.

In the case of toothed gearing, the power is transmitted entire, and the strain upon the teeth is simply equal to the power required to overcome the resistances; whereas, in the case of a belt, the tension must be greatly in excess of the driving power wanted.

181. The ratio of the adhesion to the pressure is independent of the area of the surface of contact, so that with a given pressure, we get the same adhesive grip on a pulley of 12 inches diameter, as on one of 36 inches diameter, when the belt covers an arc of the circumference

containing the same number of degrees in the one as in the other.

132. But, according as we increase or diminish this arc, we increase or diminish the compression of the pulley, and likewise the leverage of the power which is exerting the pressure, so that with a given counterbalanced pulling tension in the belt equal to 10·0 when the arc of contact is half the circumference equal to 180°, as in Fig. 14, we have the direct pressure on the pulley axle equal to 10·0, and the leverage of the pull represented by the radius at right angles to the pull, equal to 10·0 likewise. Whereas, were the belt retaining the given tension to lie in the line *x x*, Fig. 14, and merely touch the crown of the pulley, both pressure and leverage would be reduced to 0·0.

Further, were the belt to embrace the whole circumference equal to 360°, we would have the leverage of the power squared, so as to be $10 \times 10 = 100$; or, what amounts to the same thing inversely, the square root of the ratio K (which we will determine presently, paragraph 136, and which may be taken to represent the leverage of the tension), for the full arc of 360°, with a given tension, gives the value of the leverage for 180°, that is, the ratio of slack tension to the pulling tension. And, when the tensions work at an angle with the centre line of the pulleys, so as to reduce the grip, we have the working power (= Q) for an arc of 180°, equal to the working power for an arc of 90° multiplied by the square of K for that lesser arc, when the pulling tension is the same. (Paragraphs 169 and 174.)

133. We will employ the formula given by M. Prony, to find the loss in driving power when the arc is reduced; also, the ratio of tensions in the pulling and the slack lines, and the ratio of frictional adhesion to pressure.

P = Pulling tension = T. Fig. 34.

L = Load to be dragged = T′.

A $=$ Arc of the circumference.
R $=$ Radius of pulley.
C $=$ Coefficient of friction.
$\cdot 434$ $=$ Constant for all powers.

134. In the case of a power T dragging a load T′ by a rope passed over a fixed saddle A, Fig. 34, T′ is the weight that T would just balance at the point of sliding; so that, when working on a pair of pulleys free to move, the load that T is capable of balancing at the sliding point is T′ + the frictional resistance, which we will call Q.

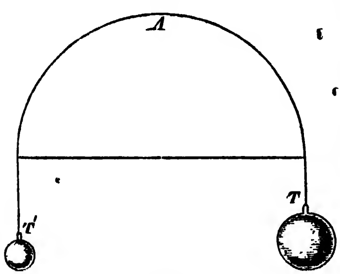


Fig. 34.

It is clear, therefore, that in the case of pulleys with free motion, T′ represents the minimum slack tension, and Q, suspended on the same side with T′, the load weight that may be held balanced at the sliding point by the frictional resistance of the belt upon the pulley, with T for power.

135. Further, $T + T' = T_1$, the joint tension of both sides in the case of pulleys in motion, which must remain a constant quantity, up to the sliding point, because, when resistance has to be overcome by an increase in the tension T, it can receive this increase only at the expense of the T′ side, by throwing slack round into it, in equivalent measure to its own increasing tension, and we have Q always equal to the variable difference between T and T′.

136. In employing the formula we will substitute T and T′ for P and L.

$$\text{Logarithm of } T = \text{Log. } T' + \left(\cdot 434 \times C \times \frac{A}{R}\right).$$

The ruling value within the brackets we will term K, so that the formula may be thus expressed: $T = T' \times K$;

and when we know the quantity T and wish to find T′, we have $T' = \frac{T}{K}$.

When we know the total tension $T + T' = T_1$, and the resistance Q to be overcome, we have $T' = \frac{Q}{K-1}$, and, subtracting T′ from T_1, we get T the tension of the pulling side; and, should T_1 be known, but Q be unknown, we have $T' = \frac{T_1}{K+1}$. (Paragraph 154.)

137. Fig. 35 represents two belt pulleys of equal diameter, say 12 inches. The arc of contact is here

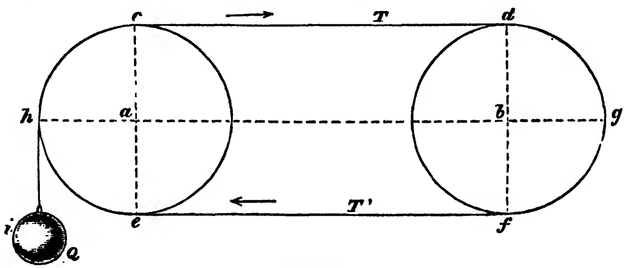


Fig. 35.

equal to half the circumference, which we will express in feet.

Let *b* be the power and *a* the resistance pulley, and let the slack tension be represented by the weight *i*, of 50 lbs; and employ the coefficient of friction for leather belting upon iron pulleys, equal to ·38 of the pressure; then, using these definite quantities, we have,

$$\text{Log. } T' + \left(\cdot 434 \times \cdot 38 \times \frac{1\cdot 5708}{0\cdot 5}\right) = \text{Log. } T.$$

Log. T′ + (·518109) = Log. T; and, as the common logarithm of T′ = 50 lbs. is 1·69897, we have 1·69897 + ·518109 = 2·21707 the logarithm of 164·82, the number of pounds in the pulling tension T.

Further, ·518109 is the log. of the number 3·296 = K for 180° contact. (Paragraph 157.)

When the pulleys are equal, $\frac{A}{R} = 3{\cdot}1416$ = the constant ratio of circumference to 1 for diameter; and it is this quantity only that is changed when, for the sake of higher speed in the driven pulley *a*, we increase the diameter of *b*.

138. Thus, let Fig. 36 take the place of 35, and let the driving pulley *k* be 3 feet diameter, and the distance *k j* be 10 feet. We have here the belt lines *o n* and *m l* converging to a point at *s*, so as to make the angle *l s n* = 11°. Now, 180° − 11° = 169°, which, divided by 360°,

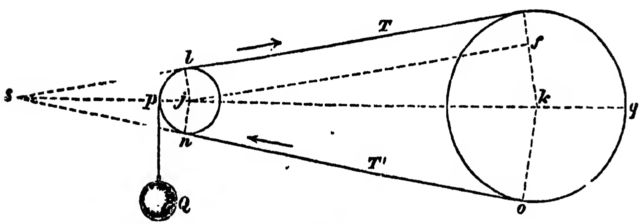


Fig. 36.

gives for the arc of contact ·47 of the whole circumference; so that, as the circumference of the pulley *j* = *a* of Fig. 35, is 3·1416, we have 3·1416 × ·47 = 1·4765 feet, the length of the arc A, represented by *l p n*; consequently, the question now stands,—

$$\text{Log.}\,T' + \left(\cdot 434 \times \cdot 38 \times \frac{1{\cdot}4765}{0{\cdot}5}\right) = \text{Log.}\,T, \text{ or } 1{\cdot}69897$$

+ ·486948 = 2·1858, the log. of 153·42 lbs. in the pulling tension T. And, ·48694 is the log. of 3·068 = K for 169° contact.

139. Reducing the distance *j k* to 5 feet, and keeping all else as before, we have the angle *l s n* = 22° 40; and 180° − 22° 40 = 157° 20 ÷ 360° = ·437 of the whole cir-

cumference; so that $3·1416 \times ·437 = 1·3728$ feet for A. Hence,

$$\text{Log. T}' + \left(·434 \times ·38 \times \frac{1·3728}{0·5}\right) = \text{Log. T, or } 1·69897$$

$+ ·45279 = 2·1516$, the log. of the number 141·76 lbs. pulling tension T.

And, ·45279 is the log. of $2·836 = \text{K}$ for 157° 20 contact.

140. When the angle $l\,s\,n$ is a right angle $= 90°$, we have $180° - 90° = 90° \div 360° = ·25$ of the circumference, and $3·1416 \times ·25 = ·7854$ feet for the arc of contact A, so that the question here stands thus:—

$$\text{Log. T}' + \left(·434 \times ·38 \times \frac{·7854}{0·5}\right) = \text{Log. T, or } 1·69897$$

$+ ·25905 = 1·95802$, the log. of the number 90·79 lbs. pulling tension T; and ·25905 is the log. of the number $1·81 = \text{K}$ for 90° contact.

141. The slack tension T′ has here been constant, and the loss of driving power consequent on the lessening of the arc from 180° is (paragraph 164)

$164·82 - 153·43 = 11·39$ lbs. for 169°
$164·82 - 141·76 = 23·06$ „ „ 157° 20′
$164·82 - 90·79 = 74·03$ „ „ 90°

SECTION X.

142. As before observed, the ratio of friction to pressure represented by the coefficient is independent of the area of contact; but the durability of the material under pressure is dependent upon the greatness of the surface to a given pressure; that is, the pressure of 1 ton on a square inch of surface brings the friction nearer to the point of abrasion than when distributed over 20 square inches.

Were the friction here produced by the pressure of one rectangular body drawn horizontally upon another, each

of those 20 square inches would be bearing 1 cwt.; whereas, in the case of a belt upon a pulley, with a tension of 1 ton, the pressure may be determined in the manner we will now state.

143. As regards the forces acting at the points *d* and *f*, of Fig. 35, we have the pressure on the axle equal to the tension, when the belt is working upon two equal pulleys; but as regards pressure acting upon the face of the circumference, we have the amount per inch length of arc as much less than is borne per inch length of radius as the arc of contact on one side of the centre line is greater than the radius.

144. For the arc of 90° in a diameter of 1 foot = ·5 foot radius, and ·7854 foot arc, we have $\frac{\cdot 500}{\cdot 7854} = \cdot 6366$ of the tension, operating in direction parallel with *g b*.

And, as 1·00 — ·6366 = ·3634, we have ·3634 of the tension operating in direction parallel with *d b*, and consequently neutralised in greater or less degree as regards side-thrust on axle according to the tension of the other side, by corresponding pressure parallel with *f b*.

Supposing the radius *d b* to measure 20 inches, and the arc *d g* 31·416 inches, we have *d g* × ·6366 = 31·416 × ·6366 = 20 cwts., and this is equal to the radius multiplied by full value: thus, 20 × 1·0 = 20 cwts.; that is, while each inch of the arc *d g* is bearing a pressure of ·63 cwt. (as the mean for the whole number of 31·416 inches contained in it), the radius *d b* is bearing a mean pressure of 1 cwt. per inch, or 20 cwt. upon its 20 inches' length *d b*, with the axle centre *a* acting as a fulcrum of leverage, and therefore supporting this weight.

145. $\sqrt{\text{sine}^2 + \text{cosine}^2}$ = radius, or the resultant of the forces acting respectively parallel with *g b* and *d b*. This, however, though showing that the pressure of the tension radiates towards the axle centre, does not determine either

the actual pressure on the axle, or the gradual diminution of the pressure from *d* to *g*, when the arc of contact is lessened; but, as the sine + co-versine, or the cosine + versine = 1·0 for the radius of all angles from 1° to 90°, and, adopting the former expression, as the co-versine must decrease as the sine increases, and as we have the pressure on the axle increasing with the increase of the sine, we have here the sine representing the power of the tension upon the arc, and the co-versine the quantity which this power is less than the full tension 1·00.

146. We will here employ Fig. 37, making the arc

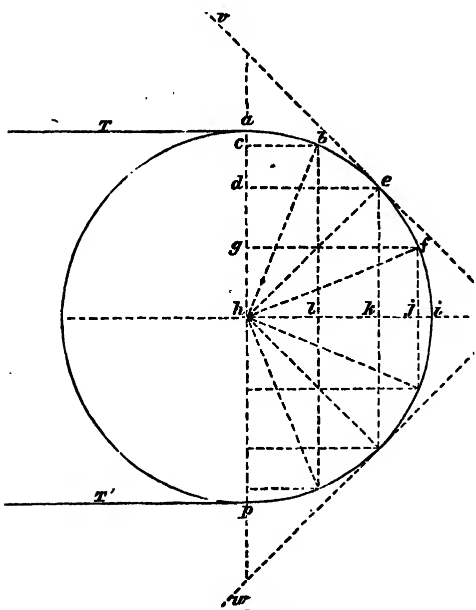


Fig. 37.

a i p of 180° equal to the arc *d g f* of Fig. 35, and making the radius 20 inches, as before.

If we make the angles open from i as 0°, we have the sines and co-versines represented by the lines parallel with $a h$; the cosines and versines parallel with $h\,i$.

In this case, with the tension a constant quantity for all arcs of contact, we have the axle pressure as the leverage length of the sine; whereas, when we make the angles open from a and p, we have the leverage as the length of the cosine. Reference to a Table of Sines, &c., will at once show that the cosine for the angle $a\,h\,e$ is identical with the sine of the angle $i\,h\,e$.

Taking the angle $i\,h\,a$, opening from i, we have the sine $h\,a = 1{\cdot}0$, and the cosine, co-versine, and versine $= 0{\cdot}0$; and when it opens from a as zero, we have the sine $h\,i = 1{\cdot}00$.

Taking the angle of $45° = i\,h\,e$, opening at i, we have the sine $= e\,k$, cosine $= e\,d$, co-versine $= d\,a$, and versine $= k\,i$.

On the other hand, if we take the complimentary angle $a\,h\,e$, opening from a, we have the cosine $= e\,k$, sine $= e\,d$, versine $= d\,a$, and the co-versine $= k\,i$.

147. Now, taking a Table of Sines, &c., and adding together the fractional quantities of the sines parallel with $h\,i$ for angles 0° to 90°, opening from a, and dealing similarly with those of the co-versines, we have them respectively 57·8138 and 33·2029; then, as sine + co-versine $= 1{\cdot}0$, we get the mean values of sines and co-versines respectively; thus, $57{\cdot}8138 + 33{\cdot}2029 = 91{\cdot}0$.

$$\frac{57{\cdot}8138}{91{\cdot}0} = {\cdot}6352 \text{ mean value of sines parallel with } h\,i,$$

and therefore representing mean pressure on the rim; so that the tension, multiplied by this mean value, gives the mean pressure per degree upon the arc $a\,i$, or mean pressure for the whole arc; but, as we have already shown that this lesser value ·6352 per unit of greater length in the arc is just equal to the full value 1·0 per unit of the lesser length in the radius, we have the tension at a for

90° exerting its full power 1·0 upon the axle, whether we regard it as distributed over the arc, or acting as in a simple case of leverage at the end a of the radius $h\ a$, or pressing directly upon the axle with force represented by the radius $h\ i = 1{\cdot}0$, which is the sine for the angle of 90° opening from a.

148. Further, $\frac{33{\cdot}2}{91{\cdot}0} = {\cdot}3648$ mean value of co-versines parallel with $h\ a$.

By the sines we have ·6352, and by the co-versines ·3648, and these added together, as remarked, equal 1·0, the full value of the tension.

149. The co-versine pressure of the arc $a\ i$ is balanced and neutralized by the corresponding pressure in the arc $p\ i$, so that the axle has the ·6352 mean sine pressure alone to bear, and, as we just now showed, 31·416 inches of arc × ·6352 value per inch, is nearly equal to 20 inches of radius × 1·0 value per inch.

The ·6366 mean value, got by dividing the arc by the radius, gives the equivalent value exactly.

150. Now, with the tension at $a = 1{\cdot}0$, multiplied for leverage by the radius or sine $a\ h = 1{\cdot}0$, of the angle of 90° $i\ h\ a$, we have the pressure on the axle simply equal to 1·0, thus $1 \times 1 = 1{\cdot}0$; $a\ h$ is thus seen to represent full leverage powers, and, as it is the sine of 90°, we have the lesser powers of the tensions for smaller angles all referable to it, and consequently represented by the respective sines.

151. With the same tension leading away in the direction $e\ v$, at an angle of 45° with the line $i\ h$, or what amounts to the same, with the angle $a\ h\ e = 45°$, we find the pressure on the axle operating in direction parallel with $i\ h$, thus,—

Tension × sine of 45° = pressure on axle, and as the sine of $a\ h\ e$ is $d\ e = {\cdot}7071$, we have 20 cwts. × ·7071 = 14·142 cwts. axle pressure. Or, we may divide the tension by the number of times the sine is contained in the radius

thus, $\frac{1\cdot0}{\cdot7071} = 1\cdot4142$, and $\frac{20 \text{ cwts.}}{1\cdot4142} = 14\cdot142$ cwts. pressure; and as the radius equals 20 inches, we have radius $\times$ sine $= 20$ inches $\times \cdot7071 = 14\cdot142$ inches length of sine, which is a burden of 1 cwt. per inch of sine, same as per inch of radius.

152. Employing the mean sine and co-versine values of the respective arcs, we have

inches. o

$90° = 31\cdot416 \times \cdot6366 = 20\cdot0$ cwts. pressure on axle, and

$45° = 15\cdot708 \times \cdot9 \quad = 14\cdot137$,, ,,

The mean value ·9 is for the aggregate sines of 45° to 90° in the angle *e h i*, the 90° ending at *i*; thus

$$41\cdot37658 + 4\cdot6307 = 46\cdot0$$

$$\frac{41\cdot3765}{46} = \cdot9.$$

153. In thus dealing with the arc of one side only, we

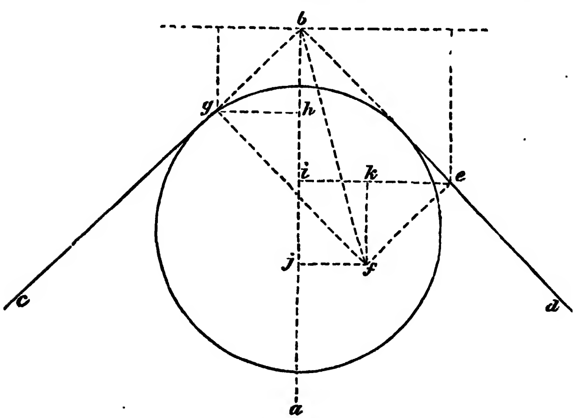


Fig. 38.

assume that there is a similar load on the other side, which would make the axle pressure twice what we have here

found it: and, should the load on the other side with the same angle be either less or more than is borne by the side treated, we may determine its pressure separately by the sine, and add the two together, or employ the graphic method of Fig. 38, in which $b\,e$ represents proportionately tension T, and $b\,g$ represents T′.

Drawing $g\,f$ parallel to $b\,e$, and $e\,f$ to $b\,g$, we have $b\,f =$ the resultant tension; and $b\,j$, the combined axle pressure of the two sides, because $b\,i$ represents the pressure due to tension in $b\,e$, and $b\,h$ the pressure due to tension in $b\,g$; then, as the angle $i\,e\,f$ is equal to the angle $h\,g\,b$, and the side $e\,f$ to $b\,g$, we have the side $f\,k$ equal to $b\,h$, so that as $j\,f$ is drawn parallel to $i\,k$, we have $i\,j = b\,h$, consequently $b\,j = b\,h + b\,i$.

154. It has been shown that when the tensions are unequal, as in the case of T and T′, Fig. 39, the tension that disappears from T in its passage round the driving wheel from d to f, is equal to the working resistance Q, so that, when the logarithmic ratio of T and T′ for the point of slipping is $=$K, we have—

$$\text{arc } d\,a = \text{K} = \text{T}$$

$$,, \quad a\,f = \frac{\text{T}}{\text{K}} = \text{T}'$$

and $\text{K}-1 = \text{Q} =$ the difference between T and T′; so that we must suppose the tension T in action from d to f; and the resistance opposed to it T′$+$Q, in action from f to d, the arcs $d\,a$ and $a\,f$, proportionately sharing Q between them.

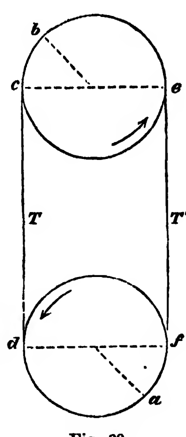


Fig. 39.

155. Now, for every lb. of frictional adhesion on the face of the arc, we have an equivalent lb. abstracted from the tension T, but the ratio K takes account of this in determining the relative proportions of T and T′.

It is clear, however, that the resistance Q will be overcome in greatest measure where the tension is greatest, in the ratio of 3·296 to 1·0 for the particular case of paragraph 137.

156. Q, on the side d, is the only unbalanced force = 1·0 at d, and 0·0 at f, because at f we have only tension T′ balancing its equivalent in T. But as Q is dependent on the joint operation of the tensions T and T′, we have T contributing to it in the ratio of 3·296 to 1·0 for T′, on the respective arcs $d\,a$ and $a\,f$; and as this ratio is for the point of slipping in direction from f to d, we have the adhesion Q in the arc $a\,f$ so much less than Q in the arc $a\,d$, that the slipping of the belt upon the pulley face, in the act of contracting to the strain T′, takes place most readily on the arc $a\,f$.

157. Thus, employing the quantities of paragraph 137, viz., T = 164·82 lbs., T′ = 50 lbs., Q = 114·82 lbs., and the logarithmic ratio K for the point of slipping = 3·296. We first find the proportions of Q for the respective arcs of T and T′, by dividing Q by K + 1, because K is here as 3·296 for T to 1·0 for T′, consequently as $\frac{T + T'}{K + 1} = T'$ we have $T - T' = \frac{Q}{K + 1}$ = the portion of Q that falls to T′, the difference between this portion and Q entire belongs to T, but is subtracted from it in the form of frictional adhesion.

$$3{\cdot}296 + 1{\cdot}0 = 4{\cdot}296.$$

$$\frac{114{\cdot}82}{4{\cdot}296} = 26{\cdot}7 \text{ lbs. of Q for the arc } a\,f.$$

$$114{\cdot}82 - 26{\cdot}7 = 88{\cdot}12 \text{ lbs.} \quad ,, \quad a\,d.$$

158. In the case of the driven pulley we have the order of the arcs reversed, so that the lesser arc $c\,b$ takes the place of the greater arc $d\,a$ in the driving pulley. This is owing to the belt stretched by the tension T passing on to the driving pulley at d, and being so held by the frictional adhesion that it reaches the ratio point a before it

begins to slide in contracting its length to correspond with the tension T′; and, to the belt with this lesser stretch of T′ passing on to the driven pulley at *e*, and being so aided by the adhesion that it is not overcome, so as to slide in stretching to the greater tension at *c*, till it reaches the ratio point *b*. We have, consequently, the stretching and contraction of the band on the respective pulleys taking place mainly on that side of the arc where the band leaves contact.

159. This sliding under alternate extension and contraction prevents the two pulleys and the belt from moving at precisely the same rate, but as the difference is dependent greatly upon the character of the belt for stretching, and is very small in amount, it may be disregarded in ordinary practice.

160. When resistance in the driven pulley becomes greater than the K ratio difference between T and T′, the belt slips, and must be tightened either by a fresh splice or by pressing the two pulleys further apart, or by employing a jockey pulley.

We will suppose the pulleys are pressed further apart with a force equal to 100 lbs., which is equally borne by both pulleys, and is equally shared by both tensions T and T′, being 50 lbs. added to each. So that

$$\begin{aligned} T &= 164{\cdot}82 + 50 = 214{\cdot}82 \text{ lbs., and} \\ T' &= \ \ 50{\cdot}0 \ + 50 = 100{\cdot} \quad ,, \\ \hline Q &= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 114{\cdot}82 \ ,, \end{aligned}$$

161. We here see that the difference between the tensions remains unchanged, but the ratio between them has been altered, thus $\frac{214{\cdot}82}{100} = 2{\cdot}148$ in place of $3{\cdot}296$ for the point of slipping.

Then $2{\cdot}148 + 1{\cdot}0 = 3{\cdot}148$ divisor to get the proportions in which Q is shared by T and T′.

$\frac{114 \cdot 82}{3 \cdot 148} = 36 \cdot 4$ lbs. for the arc *a f*, and consequently 78·42 lbs. for the arc *a d*.

The belt will not now slip till the tension T′ is again reduced to the ratio K found by the logarithmic rule. Thus $\frac{T + T'}{K + 1} = \frac{214 \cdot 82 + 100}{3 \cdot 296 + 1 \cdot 0} = 73 \cdot 3$ lbs. for T′ at point of slipping, and as $314 \cdot 82 - 73 \cdot 3 = 241 \cdot 52$ lbs. for T at the point of slipping, we have $241 \cdot 52 - 73 \cdot 3 = 168 \cdot 2$ lbs. for Q, which is

$168 \cdot 2 - 114 \cdot 82 = 53 \cdot 38$ lbs. additional power in Q.

162. When one pulley is less than the other, the belt will slip on it before the slipping point is reached on the larger, because the arc of contact in the lesser pulley, when the belt is not crossed, contains a less number of degrees than the arc of the other; the value of the slipping ratio K being ruled by the number of degrees enclosed by the arc, without regard to the diameter.

163. When the belt is crossed, the arc is practically equal on the two pulleys; the only difference being caused, in the case of a horizontal direction, by the curve of T being less than that of T′, according to the greater or less elasticity of the band.

SECTION XI.

164. We will now determine the differences for wire ropes upon an iron pulley for arcs respectively 180°, 157° 20′, and 90°. Firstly, keeping the tension T constant; and secondly, keeping the working adhesion Q constant.

(1) Tensions T and T′ parallel, 180° arc.

$K' = (\cdot 434 \times \cdot 18 \times 3 \cdot 1416) =$ Log. $\cdot 24541 = 1 \cdot 759$ number, in place of 3·296 when the band is of leather.

This quantity signifies that T is as 1·759 to 1 for T′; consequently, K′ − 1 = T − T′ = Q. (Paragraph 191.)

165. The coefficient of friction for iron upon iron dry, when there is no sliding, is ·18, but should sliding begin, the coefficient will be slightly reduced.

(2) Tensions T and T′, angle *b m h*, Fig. 37, = 11° 20, arc 157° 20

$$K'' = \left(\cdot 434 \times \cdot 18 \times \frac{1\cdot 3728}{0\cdot 5}\right) = \text{Log. } \cdot 21448 =$$

1·6385, number for ratio.

(3) Tensions T and T′, angle *e m h* = 45°, arc 90°

$$K''' = \left(\cdot 434 \times \cdot 18 \times \frac{\cdot 7854}{0\cdot 5}\right) = \text{Log. } \cdot 12272 =$$

1·3265, number for ratio.

166. We assume the tension T to be 2,240 lbs., the logarithm of which is 3·35024, from which we subtract in succession the logarithms just now found, and have in the number belonging to the reduced logarithm of the remainder, the tension T′, thus:—

				Number.	
(1)	3·35024 − ·2454	= 3·1048	=	1272·9 lbs.	= T′
(2)	3·35024 − ·21448	= 3·1357	=	1366·8 ,,	= T″
(3)	3·35024 − ·12272	= 3·2275	=	1688·5 ,,	= T‴

But we may find tension T′ directly, by dividing tension T by the number for K. Thus,

$$(1)\quad \frac{T}{K'} = \frac{2240}{1\cdot 759} = 1273 \text{ lbs.} = T'$$

$$(2)\quad \frac{T}{K''} = \frac{2240}{1\cdot 6385} = 1367 \text{ ,,} = T''$$

$$(3)\quad \frac{T}{K'''} = \frac{2240}{1\cdot 3265} = 1688 \text{ ,,} = T'''$$

167. Subtracting tension T′ from tension T, gives the weight of the frictional adhesion. Thus,

(1) 2240 − 1272·9 = 967·1 lbs. adhesion = Q′
(2) 2240 − 1366·8 = 873·2 ,, ,, = Q″
(3) 2240 − 1688·5 = 551·5 ,, ,, = Q‴

168. The total tension in the belt is equal to $T + T'$, or $T + T''$, or $T + T'''$, according to the arc embraced; and these total quantities, divided by 2, give the mean tension on each side for the state of rest when axle friction is disregarded. Thus,

lbs.

(1) $2240 + 1273 = 3513 \div 2 = 1756 \cdot 5$ each side.

(2) $2240 + 1367 = 3607 \div 2 = 1803 \cdot 5$ „ „

(3) $2240 + 1688 = 3928 \div 2 = 1964 \cdot 0$ „ „

$T' + Q' = T''' + Q''' = T = 2240$ lbs.

$T''' \div T' = K'''$, that is $1688 \div 1273 = 1 \cdot 326$.

169. The arc of T' is twice the arc of T''', hence the greater pressure required by T''' to give frictional resistance to the pull of T. But we find that, notwithstanding this greater pressure, Q''' belonging to it, is only $\dfrac{1}{1 \cdot 75}$ of Q', thus, $967 \cdot 5 \div 551 \cdot 5 = 1 \cdot 759 = K'$, and as this is the square of $K''' = 1 \cdot 326^2$, we have the power that determines the value of Q''' equal to the square root of the power that determines the value of Q', for a given constant tension T. (Paragraph 132.)

170. We will now ascertain the tensions when the resistance $= Q'$ for 180° is kept constant; employing the same angles as before.

$$\frac{Q'}{K' - 1} = T', \text{ so that}$$

$$(1) \ \frac{967}{1 \cdot 759 - 1} = \frac{967}{\cdot 759} = 1273 \text{ lbs.} = T'$$

$$(2) \ \frac{967}{1 \cdot 638 - 1} = 1516 \text{ lbs.} = T''$$

$$(3) \ \frac{967}{1 \cdot 3265 - 1} = 2961 \ \text{,,} \ = T'''$$

And, to find the pulling tension,

(1) $967 + 1273 = 2240$ lbs. $= T$

(2) $967 + 1516 = 2483$ „ $=$ „

(3) $967 + 2961 = 3928$ „ $=$ „

171. And, for mean tension in a state of rest, disregarding axle friction which, in ceasing from motion, will retain in the side T a quantity in excess of the true mean, equal to the force on the rim required to balance the friction on the axle,

lbs.
(1) $2240 + 1273 = 3513 \div 2 = 1756{\cdot}5$ each side.
(2) $2483 + 1516 = 3999 \div 2 = 1999{\cdot}5$,, ,,
(3) $3928 + 2961 = 6889 \div 2 = 3444{\cdot}5$,, ,,

172. Next, for the ratio of resistance to total tension $T + T'$ we have,

Ratio.
(1) $\frac{3513}{967} = 3{\cdot}633 = 1{\cdot}0$ Relative ratio.

(2) $\frac{3999}{967} = 4{\cdot}156 = 1{\cdot}144$,, ,,

(3) $\frac{6889}{967} = 7{\cdot}124 = 1{\cdot}961$,, ,,

173. The relative ratios here show that, approximately, the frictional adhesion equal to a given resistance is effected in an arc of 180° with about one half the total pressure $T + T'$ required in the arc of 90°.

174. When the rope makes one complete turn round a drum, we square K, and by the square product divide the pulling tension T to get the slack tension T'; and when more than one complete turn is made, we multiply K into itself as many times as there are half-circumferences in the turns, and dividing T by this square product, get T' at the free end.

175. It is equally the same whether the turns be round one drum, as in Fig. 40, or embracing two, as in Fig. 35, because, in either case, the "slack tension" T' at the close of each successive half-circumference is treated as the tension T for the succeeding half-circumference, thus, $\frac{T}{K} = T'$; $\frac{T'}{K} = T''$; $\frac{T''}{K} = T'''$, and so on.

176. Let us here see how many half-turns will be

required to take up all the weight of $T = 2240$ lbs., disregarding bending resistance.

We have already found K, for iron upon iron dry, on an arc of 180°, to equal 1·759, consequently,

$\frac{2240}{1·759} = \frac{1273}{1·759} = \frac{728}{1·759} = \frac{411}{1·759}$, and so on, till at the end of 11 half-turns = 5·5 whole turns we have $T' = 4$ lbs.

177. Now, we find that the coefficient of friction ·18 is contained 5·5 times in the whole pressure 1·0; so that an approximate idea may be had directly of the number of turns required.

Supposing the band were of leather, we have $\frac{1·0}{·38} = 2·63$ times; and employing K = 8·296 for leather on an arc of 180°, we reduce the tension from 2,240 lbs. to 6 lbs. in 2·5 whole turns, and to 1·8 lbs. in 3 whole turns, so that this direct method is near enough for ordinary purposes.

But we may here observe that ·18 for iron is employed merely as an average coefficient that will vary according to the condition of the surfaces, and will be found greater for galvanized iron or steel than for plain iron or steel.

And, in the case of leather bands on iron pulleys, the coefficient ·38 is for moistened leather; for dry leather it is only ·28.

178. We get the same ultimate result when in the formula $\left(·434 \times c \times \frac{\text{arc}}{\text{radius}}\right)$, we make the arc equal to 11 half-circumferences. Thus, taking for simplicity a 12-inch diameter, we have $\frac{3·1416}{2} = \overset{\text{feet.}}{1·5708}$ half-circumference, $\div \overset{\text{feet.}}{·5}$ radius $= 3·1416 \times 11$ times $= 34·5576 \times ·18 \times ·434 = 2·6996$, which is the logarithm of the divisor

K; the number belonging to this logarithm is 500·7, so that $\frac{2240}{500 \cdot 7} = 4 \cdot 4$ lbs. $= T'$ at the end of the 11th half-circumference.

When we know the logarithm of K for the first half-circumference, we at once multiply it by the total number of half-circumferences embraced; the product is the logarithm of the increased value of K.

In this case it is ·2454, so that

$$\cdot 2454 \times 11 = 2 \cdot 6996 \text{ Log.}$$

SECTION XII.

179. In the case of the several turns being all round one drum, as in Fig. 40, the pressure on the axle (which in this case works in fixed bearings at *a*), will be $T - T' = Q$, because T' in the line *g d* is counteracting its equivalent in T in the line *g c*, whereas in Fig. 35 it will be $T + T' + T'' + T'''$, &c., in accumulative manner, tending to bring the drums closer together.

The tendency, however, to drag away the whole frame that contains the two drums, will be just the same as for the single drum.

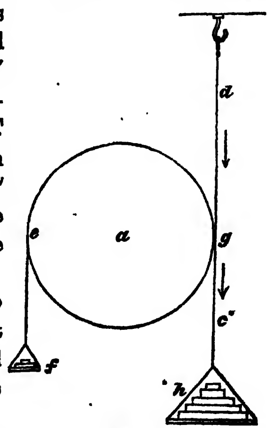


Fig. 40.

The pressure upon the axles when the pulleys are placed one above the other, as

in Fig. 41, and the load represented by the frictional resistance has to be lifted, is

$\frac{1}{2}$ (T + T') × 2 sine + load to be lifted + weight of drum. The tension for the state of rest is $\frac{1}{2}$ (T + T').

If we multiply by the sine singly, we employ the whole tension T + T' or T_1.

The sine of the angle *a h e* of Fig. 37 is *d e*; and this is equal in tabular value to the sine *e k* of the complementary equal angle *e m k* of the same figure.

180. When resistance is produced by the repression of a force acting upwards, say at *e* in the cog-wheel *g*, Fig. 41, the axle pressure at *a* will be relieved to the extent of the resistance; but the relief is at the expense of the axle *g*, as explained in reference to Fig. 28; so that we may imagine the tension T of the belt *e f* to be exerting its pressure upon both the axles *a* and *g*, the portion given to *a* being equal to T' in *c d*, and the portion to *g* equal to the resistance Q.

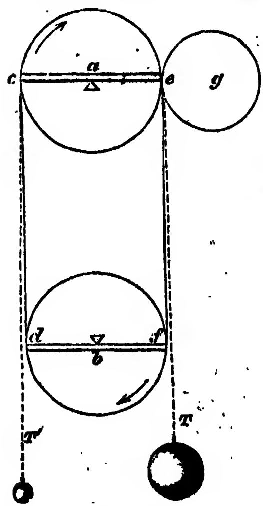


Fig. 41.

181. We will employ the tensions found for constant resistance Q to get the differences in the axle pressure between parallel and angular tension.

Thus, for parallel tension on an arc of 180°, we have, employing the quantities of paragraph 167,

(1) T + T' + Q = 2240 + 1273 + 967 = 4480 lbs. + weight of pulley = pressure on axle of top pulley.

But it is T + T' = 3513 — weight of pulley = pressure on axle of bottom pulley. We here assume that the

pulleys are working in vertical line, and that the resistance is a load to be lifted by the upper pulley.

182. Next, for an angle of 45°, in an arc of 90°, we have

$$(2)\ 3928 + 2961 = \frac{6889}{2} = 3444{\cdot}5 = \tfrac{1}{2}\,(T + T')$$

2 Sine.

$3444{\cdot}5 \times 1{\cdot}4142 = 4871{\cdot}2 + 967 = 5838{\cdot}2 +$ weight of pulley = pressure on axle of top pulley.

We here see that with the resistance constant we have had to increase the axle pressure by an amount equal to 1358 lbs., when we reduce the arc one-half from 180° to 90°.

183. When the band embraces a second arc of 180°, say on the lower pulley of Fig. 41, with the first tension $T = 2240$ lbs., we have $T' - T'' = 1273 - 723 = 550$ lbs. resistance in the second arc, and we find that this stands in the same ratio to the lesser tensions $T' + T''$, as 967 to the greater tensions $T + T'$, thus,

$T + T' : T' + T'' :: 967 : 550$; or, to treat it as expressed in paragraph 169, the square root of K for 180° employed as divisor of T', gives a ratio of tension to resistance in the second arc relatively equal to the ratio in the first.

184. Further, that $T + T'' + (T - T'') = 2240 + 723 + (2240 - 723) = 2963 + 1517 = 4480$ lbs. total tensions and resistance on the two arcs of 180°, and that this is equal to $3513 + 967$ for the first arc, and equal to the total for arcs, 1, 2, and 3, thus, $T + T''' + (T - T''') = 2240 + 411 + (2240 - 411) = 4480$, showing that the value of tension plus resistance Q is a uniform quantity, whether we take its effect upon the first arc or upon the whole of the eleven arcs of paragraph 176.

185. Supposing in Fig. 41 that the adhesive power to overcome the resistance in the pulley *a* were got by means of the band taking say 11 half-turns round the

pair of pulleys as shown, the weight T representing the 2240 lbs. pulling tension, and the weight T′ the ultimate slack tension, borne respectively by the opposite sides *e* and *c* of the upper pulley, it is clear that the pulley *a* has greater pressure to sustain from the tension, and consequently has more frictional adhesion on its rim than the lower pulley *b* has.

186. Thus, number the successive arcs of 180° each by the numerals 1, 2, 3, &c., then successively subtracting T′ from T, or as we here have it, tension 2 from tension 1, and 3 from 2, &c., to get the frictional resistance Q, and, for the total quantity Q in pulley *a*, adding together Q 1, 3, 5, &c., and for the pulley *b* adding Q 2, 4, 6, &c., we get the working grip on the respective pulleys; thus, disregarding fractional quantities,

Number.	Tensions.	Friction.	Pulley *a* Q	Pulley *b* Q
1	2240	967	967	
2	1273	550		550
3	723	313	313	
4	411	177		177
5	234	101	101	
6	133	58		58
7	75	32	32	
8	43	19		19
9	24	11	11	
10	13	6		6
11	7	3	3	
12	4			
	5180	2237	1427	810

187. We here see that the aggregate friction, inclusive of the 4 lbs. tension remaining in T′ is equal to the whole

tension T = 2240 lbs.; so that the whole of the tension T, minus 4 lbs., is here available for power.

188. We see also that the friction Q is in the ratio of K = 1·759 for the pulley *a*, to 1·0 for the pulley *b*. But, for pressure on the respective axles, we have 5180 lbs. for *a*, and 5180 − 2240 = 2940 lbs. for *b*.

189. The pressure of the tensions 2, 3, 4, &c., in the lines of rope between the pulleys, is acting equally upon the two axles; but the pressure of the first tension, No. 1, is borne by the axle of *a* only.

190. And we find that the ratio of axle pressure in *a* is as 1·759 to 1·0 in *b*; but this might be expected from the use made of the ratio K in the equations.

191. We have assumed that the resistance of the work to be done is all in the upper pulley *a*, and that the pulley *b* is free to move with the band. Notwithstanding this freedom of motion, however, the frictional adhesion on the rim of *b* is as effective for work as the adhesion on the rim of *a*; because, as frictional adhesion is dependent upon pressure, and as the pressure produced by the graduated tensions of the sides *e f* and *c d*, acts equally upon the two pulleys, tension alone being here considered exclusive of weight, we have the whole tension on the side *e f*, in the ratio of 1·759 to 1·0 for whole tension on the side *c d*. (Paragraph 164.)

Thus,	Side *e f*.	Side *c d*.
	2240	1273
	723	411
	234	133
	75	43
	24	13
	7	4
	3303	1877

and $\frac{3303}{1877} = 1{\cdot}759$ ratio.

And, as the two pulleys are in gear by means of the band, they must co-operate and move together—neglecting a certain small loss that occurs through the alternate stretching and contraction on the pulleys. But a rope so applied would have either to travel along the face of the pulley to allow of continuous motion, or be kept on one part by means of constant side pressure, unless the pulley face be grooved.

Later on we will speak on this point.

192. In computing the work performed by the tension T in the case of the endless belts of Figs. 85 and 86, we multiply the frictional adhesion Q by the speed in feet per minute, and divide by 33,000 to get the nominal horse-power. (Paragraph 201.)

We thus employ Q, because, as before observed, it is the unbalanced part of the whole tension T, and therefore alone is the weight measure of the work performed.

193. It must be borne in mind, however, that the ratios and quantities we have given relate to the point of sliding; that is, the power and resistance are on the balance, so that the slightest vibration would cause the belt to slip. In practice, therefore, the slack tension T′ must not be reduced so low as we have brought it by the ratio K, but must have a reserve force a little more than sufficient to cover irregularities which may increase the working resistance.

Moreover, as the formula that determines K is based upon the results of carefully-conducted experiments, with special new bands, the splices of which never came upon the pulleys (the motion available in the experiments being limited to the short distance between the latter), the reserve force in T′ must provide for deficiencies in the belt itself.

194. The case is somewhat different when as many turns are taken round the pulleys as leave the whole tension free for work, this being equivalent to tying

the T′ end to the pulley; but when the tension T is increased without a corresponding increase in T′, the band will begin to slide just the same as when the working adhesion is the smaller quantity Q, for a single half-turn.

Assuming that T is all free for work, we multiply it by the speed in feet per minute and divide by 33,000 to get the nominal horse-power.

195. Good new belt-leather has been found to break with an average tension of 5,000 lbs. applied quietly per square inch of sectional area.

The working tension for continuous service ought not to be more than about $\frac{1}{14}$th of this, or about 350 lbs. per square inch.

A thickness of $\frac{1}{16}$th inch, expressed in decimals, is $\frac{1 \cdot 00}{16} = \cdot 062$ inch; so that $\frac{3}{16}$ths ordinary thickness equals ·186 inch; therefore, for a breadth of 1 inch, we have $\cdot 186 \times 5000 = 930$ lbs. breaking strain, and $\cdot 186 \times 350 = 65 \cdot 1$ lbs. continuous service strain; so that, for the tension named in paragraph 187, we require a breadth of $\frac{164 \cdot 82}{65 \cdot 1} = 2 \cdot 53$ inches.

196. But with the same working tension, when we double the breadth, we reduce the strain per square inch of section to one-half the strain for the single breadth, and thereby save the belt. The axle pressure is the same, however, because the belt of double breadth is simply doing the same amount of work upon the rim of the pulley as the single breadth had to perform.

When we double the diameter, the revolutions of pulley per minute being as before, we may reduce the tension to one-half; because we have the speed at the circumference equal to 2, and this multiplied by ·5 tension = 1 power; the same as 1 speed × 1 tension = 1 power.

197. As before observed, when the two pulleys *a* and *b*, Fig. 35, are at rest, the tension on each side is nearly

equal to the mean of $T + T'$; consequently, when motion begins, the driving pulley has to stretch the pulling line *c d*, to the tension required to overcome the resistance, before the driven or load pulley can move; and in doing so, the driver is passing a corresponding amount of slack into the return side *f e*.

198. Should the resistance of the load grow less from any cause, less tension will be required to balance it, and the driven pulley will be moved by the excess of the pulling tension a fractional quantity faster than the driver, thereby throwing part of the slack of the return line into the pulling line, until the reduced load resistance and the pulling tension come to a balance; this diminishes the amount of slack on the return side.

On the other hand, should the load increase from any cause, greater tension is required; the driver must move a fractional quantity more than the load pulley to put the greater strain upon the belt, and the amount of slack in *f e* is increased correspondingly.

Hence, in a narrow belt, the return side will be slacker than when a broader belt is employed, because it will stretch more with a given tension.

199. In ordinary practice, with short belts leading direct between the pulleys on shafts in fixed bearings, it is sometimes difficult to estimate the degree of tension obtained; and it is often as difficult, without direct test, to estimate the number of units of work which have to be performed in a given time.

When the two pulleys can be thrown out of gear with all but each other by the belt, the whole tension in the two sides may be readily found by suspending a weight by a string from one of the pulleys, as the weight Q, Fig. 85. This weight, when the pulleys with belt are just on the point of motion, is equivalent to the weight of friction on the axle. Thus, let

Weight $Q = 1{\cdot}5448$ lbs.

Pulleys and belt = 50·00 lbs.
Tensions T + T′ = 164·82 + 50 = 214·82 lbs.
Radius of pulley to that of axle = 12 to 1.
Coefficient of axle friction = ·07 of the pressure.

Then, $\frac{1 \cdot 5448}{\cdot 07}$ = 264·82 lbs. and 264·82 — 50 lbs. weight of pulleys and belt = 214·82 lbs. tension = T + T′.

A weight Q suspended from the rim of the resistance pulley with the belt off, will at the point of motion give the power required to overcome the resistance; that is, will give T—T′ = Q simply.

200. Short belts require to be tighter than long ones. A long belt working horizontally increases the tension by its own weight acting in the curve formed between the pulleys.

One of the properties of this curve is to make the tension greater than is due to the simple weight of the belt; that is, greater than when the belt is hanging vertically; besides, it never loses contact.

In vertical belts, such as in Fig. 41, so little stretch is needed to make them lose contact with the lower pulley, that the tension for the state of rest requires to be greater than is found necessary for a horizontal belt, if the breadth be not increased to reduce the stretching stress per sectional square inch.

201. In paragraph 192 we have termed Q the weight measure of the work performed; but, as it may not at once be obvious how the difference between T the pulling tension and Q the work is employed, we will use Fig. 85 in explanation, and will take the tensions T and T′ for the arc of 180°, disregarding axle friction: thus—

164·82 + 50 = 214·82 lbs. (= T + T′), which remains a constant quantity for the state of rest, as well as when performing work.

This gives for the state of rest—

$$\frac{214 \cdot 82}{2} = 107 \cdot 41 \text{ lbs. tension on each side } f\,e \text{ and } c\,d.$$

The work Q is here—

$$164 \cdot 82 - 50 = 114 \cdot 82 \text{ lbs.}$$

The power to perform this work is got by transferring, by means of motion in the driving pulley at the start, one-half, that is—

$$\frac{114 \cdot 82}{2} = 57 \cdot 41 \text{ lbs. from the return side } f\,e \text{ to the}$$

pulling side $c\,d$, which increases the latter to—

$$107 \cdot 41 + 57 \cdot 41 = 164 \cdot 82 \text{ lbs.} = T.$$

And reduces $f\,e$ to—

$$107 \cdot 41 - 57 \cdot 41 = 50 \cdot 0 \text{ lbs.} = T'.$$

With 107·41 lbs. tension on each side, the forces are balanced and at rest; but on transferring $\frac{1}{2}$ Q from $f\,e$ to $c\,d$, we raise the force in $c\,d$ to Q + tension equal to what remains in $f\,e$, so that Q is the only force that is not balanced, consequently it forms the measure of the work.

The full tension T is the weight of the pressure upon the pulleys, at the points c and d, but the moving or working power requires to be equal only to Q + small force on the rim to overcome axle friction.

We must observe, however, that though Q is the effective working force, the stress in the belt on the pulling side is represented by T, and on the return side by T′, consequently we determine the breadth to bear T.

SECTION XIII.

202. With reference to Fig. 42, let—

P = Power tension, L = Load tension.
R = Rigidity of band, N = Axle pressure.
f = Coefficient of axle, r = Radius of pulley.
r' = Radius of axle.

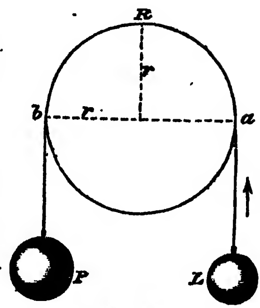


Fig. 42.

Now, as $P\,r$ is equal to $L\,r + R\,r + f\,N \times r'$, it is clear that the rigidity R requires a constant expenditure of force to overcome it. (Paragraph 219.)

203. According to Coulomb, and verified by Morin,—Resistance to bending is due firstly to natural stiffness, dependent on the particular construction of the rope, and the degree of hardness in the twist of the yarns or strands: this natural stiffness he terms A.

Secondly, to an effect of the tension in increasing the hardness of the twist: this secondary stiffness he terms B.

Making D represent the diameter of the roller, plus 1 diameter of the rope, we have the total resistance thus expressed in Bennett's Morin:

$$\frac{A + (B \times \text{tension})}{D}$$

The cords experimented with by Coulomb were of white dry yarn, and of tarred yarn.

The number of threads or yarns composing the cords are proportional to the areas: thus, a diameter of 0·0416 foot = ½ inch, contains 12 yarns, and a diameter of 0·0832 foot = 1 inch, contains 48 yarns.

The diameter for any given number of yarns may be

found by means of the following formulæ, the number being here represented by n.

Foot diameter $= \sqrt{0 \cdot 000144 \times n}$ for white dry yarns, and

Foot diameter $= \sqrt{0 \cdot 0002002 \times n}$ for tarred yarns.

For the resistance upon a drum of 1 foot diameter, of white cords, new and dry, when

R = Resistance, D = Diameter,

a = Value of resistance for single yarn,

b = Mean difference in value found by experiment for different numbers of yarns:

we have—

$$R = n \times \left[a + (b \times n) + (B \times \text{tension} \right];$$

or, with the values stated—

$$R = n \times \left[0 \cdot 002354 + (0 \cdot 001789 \times n) + (0 \cdot 001190 \times \text{lbs. tension} \right].$$

and, for a drum of different diameter, we have—

$$R = \frac{n}{D} \times \left[0 \cdot 002354 + (0 \cdot 001739 \times n) + (0 \cdot 001190 \times \text{lbs. tension} \right];$$

For tarred yarns the total resistance upon a given diameter is—

$$R = \frac{n}{D} \times \left[0 \cdot 010568 + (0 \cdot 00250 \times n) + (0 \cdot 001872 \times \text{lbs. tension} \right].$$

These formulæ are derived from experiments on new ropes ranging in diameter from 0·35 inch to 1·11 inch for white, and from 0·41 inch to 1·3 inches for tarred, and composed respectively of 6 and 60 yarns at these two extremes of each sort.

Ropes that have been some time in use are more flexible than when new; so that these formulæ do not apply in their case, and further experiment is needed to give data for precise estimate of the difference.

204. Coulomb experimented with a complete turn round a roller, as in Fig. 40, and with the tension on both sides $g\,d$ and $g\,c$ nearly equal, produced by a weight h, the

roller being rolled upon the rope by the slow action of the weight *f*, suspended by a thin line from the rim at *e*.

For steadiness there were two ropes, one at each end of the roller, so that the weight *h* was double the tension on each rope.

This was a very convenient arrangement for determining the rigidity under varying tensions; but for very stiff ropes we may get approximate results for ordinary practice from the simpler arrangement of Fig. 42.

The load *h* in Coulomb's, or rather in Amontin's arrangement, does not move; so that the effect of increased motion upon the resistance can be ascertained in simple manner; whereas, in Fig. 42, the force necessary to overcome the inertia of the two weights P and L, would have to be allowed for when starting into motion, because the resistance of this inertia is additional to the bending resistance; but, as it has been found that the resistance of friction is practically constant for different velocities, it is sufficient to bring it just to the point of yielding.

205. There has been no similar series of experiments to establish a precise rule applicable to wire ropes of different descriptions. We will here simply assume, therefore, that the rigidity or bending resistance of a particular wire rope on a given diameter—say of 1 foot, to form a standard by which the greater or less resistance on different diameters may be determined—is found approximately by suspending a load equal to the required working tension T to each of the two ends of a length hanging from a pulley, as in Fig. 42; then adding to one side a weight which, when it brings the pulley with its loads to the point of motion, is the measure of the bending resistance. Should motion be started, but so slowly as to be practically uniform, we may disregard the inertia of the loads, but subtract from the additional weight representing the bending resistance, the force on the rim required to overcome the friction on the axle.

206. The bending resistance thus got, however, is for the given diameter of pulley, and the tension of the loads employed; so that, according to Coulomb, as the bending resistance varies in the inverse ratio of the diameter, we reduce it one-half when we double the diameter; and double it when we reduce the diameter one-half.

207. With less tension the resistance is less, because the factor of tension-stiffness, represented in the case of hemp ropes by B of the formulæ, is then multiplied by a less quantity previous to its addition to the factor represented by A, which is constant for all tensions, and changes only with the diameter of the pulley, the diameter of the rope, and tightness of the twist; but the value given to it in the formulæ is for rope of the particular description used in the experiments.

B was found approximately proportional to the number of yarns in the rope, and consequently to the sectional area of the rope, and is evidently the expression of the greater tightness of the twist produced by the stretching of the rope's length by the load or tension; so that its initial value must be affected by changes in the diameter of the pulley and the tightness of the twist in the free state.

SECTION XIV.

208. In the absence of precise values for A and B, applicable generally to wire rope, the hardness of wire not being a constant quantity, whereas the yarn of Coulcomb's cord was of uniform character, we will assume, in the following practical question, that the rope is composed of 60 white dry yarns, and is at work with an arc of contact = 180°, upon a roller of wood, of ·284 foot = 3·4 inches diameter, with the coefficient of friction = ·50 of the pressure.

We will suppose that the bending resistance is 30 lbs., and will employ a load equal to this on each side.

The two loads combined, when disposed to form the tensions T and T′, for work to be done, have an adhesive power equal to 39·34 lbs., which we find thus, employing the formula of paragraph 136.

$$\cdot 434 \times \cdot 50 \times 3{\cdot}1416 = \text{Log.}\ \cdot 68172 = \overset{\text{Number.}}{4{\cdot}806} = \text{K}.$$

Then $\frac{60 \text{ lbs.}}{\text{K} + 1} = \frac{60}{5{\cdot}806}\ 10{\cdot}33 \text{ lbs.} = \text{T}'$, for the side $b\,e$, Fig. 43, so that

$60{\cdot}0 - 10{\cdot}33 = 49{\cdot}67$ lbs. $= \text{T}$ for the side $c\,f$, and $49{\cdot}67 - 10{\cdot}33 = 39{\cdot}34$ lbs. $= \text{Q}$. (Paragraph 219.)

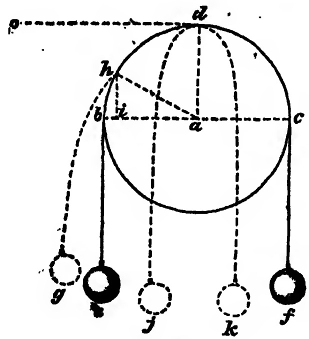


Fig. 43.

209. As before observed, paragraph 201, this free force Q, is got by transferring $\frac{1}{2}$ Q from the return side of the belt to the pulling side; so that in the present case we have $\frac{1}{2}$ Q $= 19{\cdot}67$ lbs., and $30{\cdot}0 - 19{\cdot}67 = 10{\cdot}33$ lbs. $= \text{T}'$; consequently, as the resistance to bending alone requires a tension of 30 lbs. at each side to make the rope cling close to the pulley on the whole arc of 180°, it follows that the abstraction of 19·67 lbs. from the side $b\,e$ will allow the rope to recover an equivalent amount of spring so as to take the curve $h\,g$; and as the bending

force A is a uniform quantity per unit of length in the arc of contact, we might, were the force A alone in question, at once divide the arc $d\,b$ into 30 equal lb. parts, and subtract therefrom 19·67 lb. parts to get the point h; but the decrease that has taken place in B, owing to the reduction of tension on the side $d\,b$, has reduced the total bending resistance $A + BT$; so that 19·67 lbs., minus the difference between B at 30 lbs. and at 10·33 lbs. tension, will be the quantity to subtract from the 30 parts of $d\,b$ to find the point h.

Employing the rule of paragraph 203, we have for the rope of 60 yarns—

$$R = 60 + \left[0{\cdot}002354 + (0{\cdot}001739 \times 60) + 0{\cdot}001190 \times 30\text{ lbs.}\right].$$

This is equal to $60 \times 0{\cdot}14239 = 8{\cdot}5436$ lbs. resistance for the 30 lbs. tension.

Substituting 10·33 lbs. in the room of the 30 lbs., we have $60 \times 0{\cdot}11898 = 7{\cdot}1388$ lbs.

210. But as these two quantities are for a roller of 1 foot standard diameter, whereas the roller here employed is only ·284 foot, we divide them by the latter diameter; thus,—

$$\frac{8{\cdot}5436}{{\cdot}284} = 30\text{ lbs. tension};$$

$$\frac{7{\cdot}1388}{{\cdot}284} = \frac{25{\cdot}13}{4{\cdot}87\text{ lbs. difference.}}$$

So that $19{\cdot}67 - 4{\cdot}87 = 14{\cdot}80$, which we now subtract from the 30 lb. parts of $d\,b$ to get the point h; or, as we before found, paragraph 150, that the power of the tension is as the sine of the angle; and as we have the tension acting on the arc $d\,h$, and we have here to find the loss of power, we have that loss represented by the sine $h\,i$ of the angle $h\,a\,b$, and ascertain the sine thus—

$$\frac{14{\cdot}80\text{ lbs.}}{30\text{ lbs.}} = {\cdot}493 = \text{sine of } 29^\circ\ 33'.$$

Now, treating the reduced arc *h d c*, as in paragraph 139, we have—

$180° - 29° 33' = 150° 27' \div 360° = \cdot 417$ of the whole circumference, which is here equal to—

Diameter. Circumference.

$\cdot 284$ foot $\times 3 \cdot 1416 = \cdot 8922$ foot $\times \cdot 417 = \cdot 3720$ foot arc *h d c*; so that, the radius being $\cdot 142$ foot, we have—

$\cdot 434 \times \cdot 50 \times \dfrac{\cdot 3720}{\cdot 142} = \cdot 56854$, which is the logarithm of the number $3 \cdot 70 = K$.

Then,—

$$\frac{T_1}{K+1} = \frac{60 \text{ lbs.}}{4 \cdot 7} = 12 \cdot 76 \text{ lbs.} = T'.$$
$$\text{and } 60 - 12 \cdot 76 = 47 \cdot 24 \text{ lbs.} = T.$$
$$47 \cdot 24 \div 12 \cdot 76 = 34 \cdot 48 \text{ lbs.} = Q.$$

211. We have here, therefore, in the springing of the rope to *h*, a loss of $39 \cdot 34 - 34 \cdot 48 = 4 \cdot 86$ lbs. in the working adhesion Q, caused by our having to transfer $49 \cdot 67 - 47 \cdot 24 = 2 \cdot 43$ lbs. from the working side *d c*, to the return or slack tension side *d b*, to prevent slips on the reduced arc. (Paragraph 219.)

This $2 \cdot 43$ lbs. is doubled in effect on the latter side, because it has been abstracted from the working balance in the side *d c*.

212. With the whole tension 60 lbs., we cannot in this case, on an arc of 180°, reduce the tension T' below $10 \cdot 33$ lbs. without slipping; but, if at that minimum tension, the bending resistance be only 10 lbs. on a diameter of $\cdot 71388$ foot, we get the whole working power $Q = 39 \cdot 34$ lbs., and the rope does not spring, because in T' there is $10 \cdot 33 - 10 \cdot 0 = \cdot 33$ lb. tension in excess of the bending resistance.

213. Assuming that a tension of 30 lbs. is required to bend the rope on an arc of 90° *d b*, we have this tension just balancing the resistance of the rope to bending; but

were the pulley removed and a single fulcrum placed at *d*, the weight would be found greater than the resistance, so that the rope would take the sharper curves *d j* and *d k*, showing that the 80 lbs. resistance on the given diameter has reference to an arc less than 90°; that is, the arc *d b* forms a succession of leverage distances with fulcrum

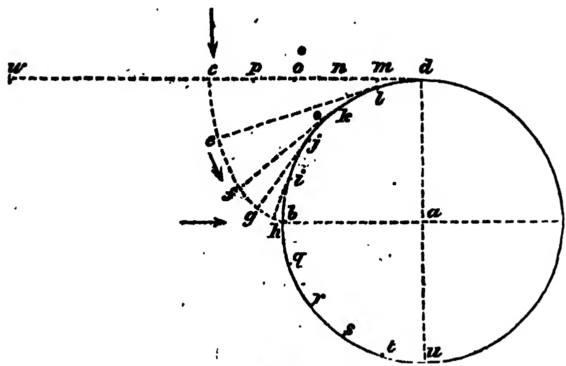


Fig. 44.

points at *l*, *k*, *j*, &c., supporting the rope in the manner shown in Fig. 44.

214. The factor A is never alone in a case of tension, but as the resistance A + B was found by Coulomb to vary with the diameter nearly, we shall treat the case as if the resistance were all of the nature of A, that is, of simple bending.

Divide the arc *d b* of 90° into 5 equal parts of 18° each, at the points *l*, *k*, *j*, and *i*, then make the horizontal line *d c* equal to *d b*, and divide it likewise into 5 equal parts at the points *m*, *n*, *o*, and *p*.

215. Let us assume for simpler illustration that the line *c d* has no weight of itself, but that a pressure of 80 lbs. is exerted at *c* in the direction of the arrows constantly perpendicular to the tangent; we will then have

the point c sinking till it coincides with b at the lower point of the arc, with the intermediate points p, o, n, and m, coinciding with the points i, j, k, and l. But, on reducing the pressure to 24 lbs., c will rise to h; a further reduction to 18 lbs. will raise it to g; at 12 lbs. it will rise to f; at 6 lbs. to e; and with all pressure removed it will resume its straight line $c\,d$, showing that the full pressure of 30 lbs. is required not for the parts nearest d, but for the final close of the bend at b.

The arc in contact with the rope at these successive reductions is

30 lbs.	$=$	$d\,b$	$=$	90°
24 ,,	$=$	$d\,i$	$=$	72°
18 ,,	$=$	$d\,j$	$=$	54°
12 ,,	$=$	$d\,k$	$=$	36°
6 ,,	$=$	$d\,l$	$=$	18°
0 ,,	$=$	d	$=$	0°
90 lbs.	$=$	1 lb. per degree.		

And as the points p, o, n, m, coincide with i, j, k, l, we get a like result to the foregoing when, in place of reducing the pressure at c, we keep the 30 lbs. constant, but shift it to the successive points p, o, n, and m; showing that the resistance A within the rope is a uniform quantity per unit of length.

Further,

				Parts.		Bending moment.
30	lbs.	at	m,	1	$=$	30
or 15	,,	,,	n,	2	$=$	,,
,, 10	,,	,,	o,	3	$=$	,,
,, 7·5	,,	,,	p,	4	$=$	,,
,, 6	,,	,,	c,	5	$=$	,,

will each bend this particular rope on the arc $d\,l$, the point c being here carried down to e.

To bend it on the arc *d k*, the pressure at the respective points requires to be

	Parts.		Bending moment.
30 lbs. at *n*,	1·0	=	30
or 20 „ „ *o*,	1·5	=	„
„ 15 „ „ *p*,	2·0	=	„
„ 12 „ „ *c*,	2·5	=	„

the point *c* is here carried down two parts to *f*, hence the 12 lbs. for *c*. The 30 lbs. pressure is required at *n*, not for the first unit *d l*, but for the double unit *d k*.

216. Each of the 5 equal parts of the whole arc *d b* takes $\frac{1}{5}$th of the whole pressure, but, when the bending force is at the outer end *c* of the line, 30 lbs. pressure on reaching *h* is no more effective for bending on the lower part *i b* with the shorter leverage *i h*, than is 6 lbs. for the first and upper part *d l*, with the greater leverage *d c*.

Dividing the arc *d b* into a greater number than 5 equal parts, does not affect the result as regards the total resistance, because, though the smaller part will require for bending less than the 6 lbs. at *c*, for the greater part *d l*, we have this advantage neutralised by the shorter leverage at the close, so that the 30 lbs. at the end *c* is still required to complete the bend to *b*.

217. And, as the angle *h i b* is the same as the angle *m d l*, the angular forces in the case of bending are of uniform value for all points of the arc *d b*.

Dividing the lower arc *b u* into 5 equal parts at the points *q*, *r*, *s*, and *t*, and lengthening *d c* to *w* to make it equal to the arc *d b u*, we find that 30 lbs. is required at *w* to bend the rope completely round to *u*, the pressure being in excess of the bending resistance till the close of the bend in the unit *t u*.

218. The leverage *d w* is twice *d c*, but there are twice the number of equal parts on the rim, with the moment of bending force equal to 30 for all alike.

And, as just now observed, this bending force is a constant quantity for all points and for any small part of the arc $d\,b$.

Thus, if we divide the arc in 90 equal parts, we still require 80 lbs. to bend $d\,c$ round to b; but, if $d\,l$ be only $\frac{1}{90}$th of $d\,c$, we have $\frac{80}{90} = \cdot 888$ lb. required at c to bend from d to l, and consequently,

$$d\,l = 90 \times \cdot 888 \text{ lb.} = 80 \text{ lbs. bending moment.}$$

219. Thus, for this particular rope, on an arc of 180° on the given diameter, as the bending resistance is a constant quantity of 80 lbs., we have T and T′ each necessarily exerting force equal to this; and this force being the same in both, affects T and T′ similarly to the increased tension of paragraph 160.

And, as the spring of the rope tends to raise it from contact with the pulley face, it is clear that the force to resist the spring must be additional to the frictional adhesion Q. (Paragraph 202.)

The bend upon the driving pulley is made on the side T; and upon the driven pulley on the side T′; consequently, each has to overcome the bending resistance once on its own side in addition to its own work.

On the driving pulley, T′ merely holds firm the bend formed by T at d, Fig. 89, till the limit of the arc of contact at f is reached, where the bending force is liberated to allow of the straight course from f to e, and is recovered at e for the new bend there.

T merely holds firm the latter bend till liberated for the straight lead from c to d, but we say simply that T and T′ in their respective straight lines $c\,d$ and $f\,e$, are each taxed to the extent of the bending resistance, so that, as in paragraph 93, we multiply this single resistance by the number of bends or pulleys to find the whole demand upon the driving power.

As the resistance R is not taken into account in K of

the formula regarding tension, we express this question of bending resistance thus:

$$Q + R + \text{axle friction} = \text{burden on power.}$$

In the case of Fig. 42, as the bend is formed by T′ at *a*, and simply liberated at *b* of this figure, we might suppose that T has no active resistance to overcome; but, as the force required to hold the bend is equal to the force at the point of formation, R must be contained in both.

In ordinary leather belts, on large pulleys, the bending resistance is so small that it may be disregarded, but it must be taken into account in the case of stiff rope bands, where, to save both band and journals, the whole tension $T + T'$ is wanted as near as possible to the requirements.

220. Supposing the diameter were doubled, we have the arcs *d b* and the horizontal line *d c* of Fig. 44 doubled in length, consequently with the doubling of the leverage we have the given pressure at *c* exerting twice the power, so that, in this case, 15 lbs. at *c* would bend round to *b*, and

$$15 \text{ lbs.} \times 2 \text{ length} = 30 \text{ lbs. bending moment as before.}$$

On the other hand, were the diameter reduced to one-half, we would have the leverage of the pressure at *c* reduced to one-half, so that, as pressure by half distance equals half moment, the pressure at *c* must here be 60 lbs. to bend round to *b*, and we have 60 lbs. $\times \cdot 5$ length $= 30$ lbs. (See paragraph 309.)

SECTION XV.

221. Ropes of hemp or wire are often employed for driving bands. Their resistance to bending is greater than that of flat leather belts, and as the surface in contact with the pulley is less, the pressure per square inch of

actual contact must be greater, and therefore more severe upon the material.

This, however, does not affect the amount of tension required for work, because, as friction is independent of the extent of surface, we get the same driving power from 10 lbs. pressure or tension on the narrow line of contact with the pulley in the case of a circular rope, that we would get from the same pressure supposing the rope flattened out so as to have a surface of contact many times greater.

222. When we know the weight per foot of a long belt or rope working horizontally, as in Fig. 45, we find the

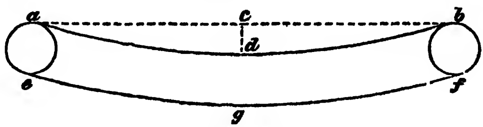


Fig. 45.

tension in the curve $a\,d\,b$ by multiplying the whole weight of the part between a and b by the horizontal distance $a\,b$, then dividing the product by 8 times the deflexion $c\,d$;

$$\frac{\text{whole weight } a\,d\,b \times a\,c\,b}{\text{deflexion } c\,d \times 8} = \text{tension at the points of sus-}$$

pension a and b. This rule, however, applies only to curves in which the deflexion is small compared with the span; so that, the flatter the angle of suspension, the closer the approximation.

223. The tension on the lower curve $e\,g\,f$ will, of course, be nearly the same as $a\,d\,b$ in the state of rest, when the axles of the pulleys are free to move for adjustment of the tension; so that, supposing $e\,g\,f$ to be the return line, the driving power has simply to transfer tension from $e\,g\,f$ to the pulling line $a\,d\,b$, paragraph 201, until the tension of the latter, minus what remains in $e\,g\,f$, is sufficient to overcome the load resistance, in the manner explained with reference to Figs. 34 to 41.

224. When a very long band works in an angular direction, as in Fig. 46, we must, in estimating the forces, employ the angle which it makes in leaving the upper pulley at h; unless the curve $h\,i$ be such that if continued beyond i, so as to hang freely from h to n, the lowest point were found at i, so as to show that $h\,i$ formed one-half of

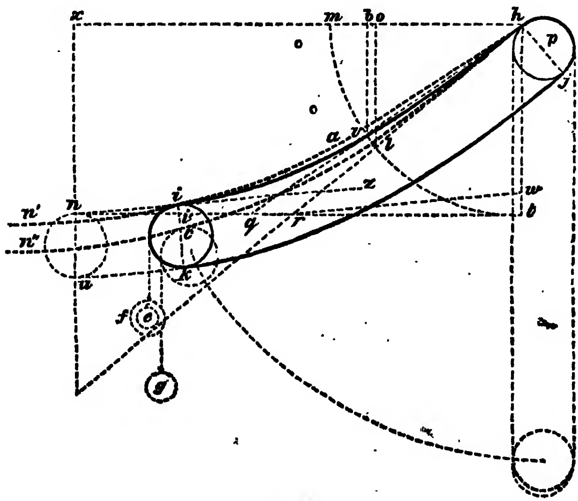


Fig. 46.

a full catenary curve; in which case the rule just now given would apply, because the tension at h would be the same as for a complete curve; but it would be convenient to express the rule for the half-span thus:

$$\frac{\text{whole weight of } h\,i \times \frac{1}{2} \text{ span } h\,x}{\text{deflexion } x\,i \times 4} = \text{tension at } h.$$

225. The band at each of the points h, i, j, and k, that is, where it first touches the pulleys, forms a tangent to the pulley circle, and must therefore start at right angles to a radial line drawn from the point of contact to the

pulley centre; so that the line $h\ r$ is always at right angles to $h\ p$.

This, however, has no direct concern with the angle of suspension, because, in finding the value of the angle at h, in relation to the curve, we take it in relation to the horizontal line $h\ x$, that is, we find the number of degrees in the angle $x\ h\ l$.

226. From a table we get the sine of this angle, represented by the vertical line $l\ o$, by which we divide the weight of the band $h\ i$, and have in the quotient the tension at the point h.

As we will explain presently, the sine $l\ o$ may be drawn at any distance from h without affecting its value in relation to the angle, but it must always be proportionate to the tabular radius.

The tension at j is found similarly.

The rule for Fig. 46 is thus expressed:

$\frac{\text{weight of } h\ i}{\text{sine of the angle } x\ h\ l} = \text{tension at } h$; so that, if the weight of $h\ i$ equal 100 lbs., and the angle $x\ h\ l = 40°$, the sine of which is ·64278, we have $\frac{100}{\cdot 64278} = 155\cdot 57$ lbs. tension at h. (Paragraph 270.)

227. Or, instead of dividing by the sine, we can multiply by the cosecant $h\ r$, and get the same result; thus, the cosecant of 40° is 1·5557, consequently $100 \times 1\cdot 5557 = 155\cdot 57$ lbs. tension at h, as before.

228. Sines, cosecants, tangents, &c., as given in tables, are merely the relative proportions which the sides of a triangular figure bear to one another, when the length of one of these sides is employed as the radius to represent any given quantity which must stand in the tables as a whole number 1·00; the other sides being fractionally greater or less than 1·0 according to the angle.

We will here put into visible form the values of the sine, &c., of the angle $x\ h\ l$.

Let the vertical line $h\,t$, in Fig. 46, represent the 100 lbs. weight of rope $h\,i$, and from the centre h draw the circular arc $t\,l\,m$; it is clear that $h\,m$, $h\,l$, and $h\,t$ are equal, being radii of the same circle.

From l draw a line $l\,o$ at right angles to the horizontal line $m\,h$, and from t draw a horizontal line to cut the continuation of the angular line $h\,l$ at r. We thus have formed two triangles, viz., $h\,l\,o$ and $h\,r\,t$, the respective values of which we will now explain. (Paragraph 237.)

229. The values of the sides of the triangles are simply relative, and are ruled by the enclosed angles, without reference to the space enclosed; hence, as by construction between the horizontal and the vertical lines $h\,m$ and $h\,t$, enclosing 90°, with the diagonal $h\,r$ common to the two divisions, the angles of the triangle $o\,h\,l$ are equal to the angles of the triangle $h\,r\,t$, and the sides of the former are to one another relatively in the same proportion as the sides of the latter.

The lines $h\,m$ and $t\,r$ are horizontal and parallel; the lines $o\,l$ and $h\,t$ are vertical and parallel, consequently the angles at t and o are right angles, and therefore equal; the angle $o\,h\,l$ is equal to the angle $h\,r\,t$, and the angle $o\,l\,h$ is equal to the angle $t\,h\,r$.

230. The side $o\,l$ in relation to the other two sides of its own triangle is proportionately equal to the corresponding side $h\,t$ of the greater triangle, and $t\,r$ to $h\,o$; while, as regards the diagonal line, $h\,l$ is proportionately equal to $h\,r$; so that, if we make the length of either of the two vertical lines $o\,l$ or $h\,t$ represent the weight of the rope $h\,i$, the lengths of the other two sides give proportionately the angular pull in the direction $h\,r$, and the horizontal pull in the direction $h\,m$ or $t\,r$. (Paragraph 235.)

We take the vertical side $h\,t$ of the greater triangle to represent the weight, so that $h\,t$ as the radius of the arc $t\,l\,m$ is termed 1·00, and, having chosen $h\,t$, we must employ the triangle in which it occurs. (Paragraph 267.)

We will in due course explain further how this makes no difference to the results, as it is only a choice between parts and counterparts.

231. The vertical angle $t\ h\ r$ is here equal to 50°; the secant of this angle, which is the same as the cosecant of the horizontal angle $m\ h\ l$, and is represented by the side $h\ r$, is 1·5557, which, proportionately, is the tension of the band at h, the simple deadweight being 1·00. The tangent of 50° represented by the side $t\ r$ is 1·1917.

The weight of the band being 100 lbs., represented by $h\ t$, we have here visibly the vertical pressure acting at h, and therefore borne by the axle of the upper pulley, equal to 100 lbs.; the diagonal pull in the direction $h\ r$ is $100 \times 1·5557 = 155·57$ lbs., as before, when employing the horizontal angle; and the horizontal draw in the direction $t\ r$, but in reality acting at h in the direction $h\ m$, is $100 \times 1·1917 = 119·17$ lbs.

232. Now, the bracket or framing employed to carry the axle of the upper pulley would have to be constructed to resist the pressures acting in these three different directions, and due allowance would have to be made for the additional weight of the pulley and the return line of band, the tension in which would be less than on the pulling side by the amount of the load resistance.

233. The slack thrown over from h to the return side at j makes the angles at j of lower value than those of h, but the manner in which the value is lowered depends upon the character for stretching.

Thus, in a band that does not stretch at all (though, as all bands stretch more or less, we merely suppose the case), the whole of the difference must come from difference of angle, which would reduce the arc of contact; whereas when it stretches freely, the contraction or lesser stretch in the return side being in the same ratio to T′ as the greater stretch is to T, we may treat the angles at j and h as practically equal for short spans.

SECTION XVI.

284. The diagonal *h r*, as already remarked, is the secant of the angle of 50° *t h r*, and is also the cosecant of the angle of 40° *m h l*.

This difference in name without a difference in value concerns the question of complementary angles, about which we will speak presently, paragraph 264, when we have given a more circumstantial explanation of the sine *o l*, &c.

285. We may first remark that all the forces at work are centred in the point *h*, and that necessity alone in the construction of the diagram places the horizontal pull-line *t r* so far away from *h*. We have only to imagine the remote sides *t r* or *o l* to be radiating from the centre *h*, in common with the other sides, to have the action of the three forces of the respective triangles rendered clearly apparent; and, as in this case of all radiating from *h*, we retain the relative lengths that the lines possessed when forming the triangles, and have the angular directions the same, the values are in no wise affected, because, as before observed, the value is ruled entirely by the angle, and not by space enclosed.

286. It does not matter whether we show the forces drawing or pushing; but as pushing forces are more easily explained, we will employ them in Fig. 47; and, as we are merely giving reasons for previous determinations, we will employ the 100 lbs. weight of band *h t* acting vertically, and the 119·17 lbs. horizontal tension *t r*.

Thus, let *y h* equal *t r*, and *v h* equal *h t*, each pushing in the direction of its length as indicated by the arrows.

We will suppose them to be in motion, and that the power of each is expended in moving a distance equal to

its own length, both reaching the end of their length at the same time.

Were the forces equal, it is clear that *v h* would have an equal effect in directing the motion downward, to *y h* in directing it horizontally, and the resultant *h r* would be at an angle of 45°; but, being unequal, *y h* directs the motion as much beyond that middle angle as its power exceeds that of *v h*; so that by the time *v h* has expended

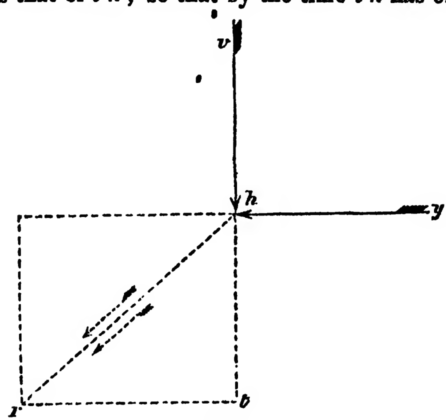


Fig. 47.

itself in moving vertically the distance of its own length equal to *h t*, *y h* has pushed it along horizontally, until the centre of resistance *h* reaches the level of *t* at the point *r*. The diagonal line *h r* is therefore the resultant of the forces in the two lines *v h* and *y h*; its length is equal to the joint angular force of these two lines, and therefore of *h t* and *t r*, which are equal to them. And as the resultant, when *h t* is the radius with the centre at *h*, is equal to the cosecant of the angle with the horizontal, as *o h l* of Fig. 46, or to the secant when the complementary angle *r h t* is employed, the cosecant or the secant, as here indicated, gives the measure of the resultant tension.

Expressed in algebraic form, the resultant is equal to $\sqrt{h\,t^2 + t\,r^2} = h\,r$; that is, equal to the square root of $h\,t$ and $t\,r$ squared, and added together. Thus, for the vertical angle $t\,h\,r$ of 50°, $h\,t$ is the radius equal to 1·0, the square of which is

$$1 \times 1 = 1{\cdot}0;$$

$t\,r$ is the tangent to the circle at t, equal in tabular value to 1·1917, which, multiplied by itself, gives

$$1{\cdot}1917^2 = 1{\cdot}419.$$

We thus have $\sqrt{1{\cdot}0 + 1{\cdot}419} \stackrel{\circ}{=} \sqrt{2{\cdot}419} = 1{\cdot}5557$, the tabular value of the secant $h\,r$.

We have employed motion in the lines $v\,h$ and $y\,h$ simply to make the description easier. When the forces consist of stationary pressure or tension, the resultant is found in the same manner, quantity and direction of the pressure or tension standing in the room of the length and direction of the motion.

237. Now, transferring these lines and angles to Fig. 48, and making $h\,t$ the radius, describe the arc $t\,l\,m$. Then drawing $o\,l$ parallel to $h\,t$, and $l\,x$ parallel to $m\,h$, we have $o\,l$, the sine of the horizontal angle $m\,h\,l$, standing in the same ratio to the angular radius $h\,l$ as $h\,t$ to $h\,r$; and we have $l\,x$, the cosine of the angle $m\,h\,l$, in the same ratio with $h\,x$, which is by construction equal to $o\,l$, as $t\,r$ to $h\,t$.

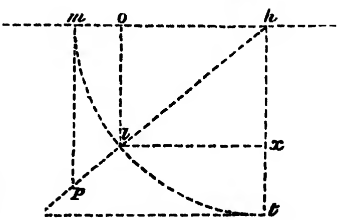


Fig. 48.

Further, $\frac{h\,t}{h\,x} = \frac{h\,r}{h\,t}$, so that as the sine of 40° is ·64278, we have $\frac{1{\cdot}00}{{\cdot}64278} = 1{\cdot}5557$.

When $h\,l$ coincides with $h\,t$, there is no angle, and the

tension at h is simply equal to the deadweight, whereas to raise $h\,l$ so as to nearly coincide with $h\,m$, the horizontal force $y\,h$ of Fig. 47, has to be so many times greater than the mere deadweight, that the band may break. On account of this, a band cannot be pulled straight in a horizontal line between two pulleys that are any distance apart.

238. Let us make the angle $m\,h\,l$ equal to 5°. In the Table we find that the sine of this angle is 0·08715, which is the length of $o\,l$ or $h\,x$ when $h\,t$ is equal to 1·00.

Now $\frac{1\cdot00}{\cdot08715} = 11\cdot47$ times $h\,x$ is contained in $h\,t$, consequently, 100 lbs. $\times$ 11·47 = 1147 lbs. tension at h in the direction $h\,l$.

The tabular value of the cosecant of the angle of 5° is ust 11·47, when the radius $h\,t$ is equal to 1·00, so that we may at once find the tension by multiplying the weight by the cosecant, or, by dividing it by the sine; when we employ the latter we get

$$\frac{100\cdot00 \text{ lbs.}}{\cdot08715} = 1147 \text{ lbs. tension at } h.$$

When the angle $m\,h\,l$ is 1°, we have the cotangent = 57·298, which, multiplied by 100 lbs. deadweight, equals 5729·8 lbs. tension at h.

SECTION XVII.

239. In a catenary curve, which is the form invariably assumed by any perfectly elastic body of uniform size and substance, such as a rope or chain or belt hanging freely between two distant points, the horizontal tension is a uniform quantity for all points of the curve; so that the horizontal line $t\,r$, is the measure at the lower pulley as well as at the upper; but, the vertical tension at any

point is simply the weight of band between that point and the lowest point in the curve; so that, for any point close to, but not quite at the lowest point in the hollow of a complete curve, the vertical tension is so small that the resultant *h r* differs very little from the horizontal line of tension *t r*.

240. We will assume that *h i* is less than half a full curve. The angle of tension at *i*, Fig. 46, is in the line *i z* at right angles to a line drawn from the point where the band first touches the pulley rim at *i*, to the centre of the axle *c*; that is, the band where it first touches the pulley forms a tangent to the circle of the rim.

From *r* in the triangle *h r t*, we draw the line *r w*, parallel to *i z*, to enable us to ascertain the proportionate strain at *i* in relation to the strain at *h*.

241. Now, when *h r* represents the tension at *h*, the tension at *i* is represented by *r w*, and as the horizontal line *t r* is common to both, because of the horizontal tension being uniform for all points of the curve, we have the vertical tension at *i* represented by the fractional length *t w*; thereby reducing the vertical tension at *h* from the full weight of 100 lbs., represented by *h t*, for the half curve *h i n*, to the smaller weight represented by *h w*.

Let that smaller weight *h w* be equal to 90 lbs., which shows that *t w* has cut off $\frac{1}{10}$th or 10 lbs. from the weight of the full half-curve.

The angle of suspension *t h r* being 50° as before, the tabular or ratio values of the secants, tangents, &c., remain unchanged; but, as they have to deal now with a reduced weight of band, we get reduced results.

Thus, multiplying the tabular value 1·5557 of the cosecant *h r*, by 90 lbs., we have 140·013 lbs. tension at *h* as the resultant strain of the reduced weight, in the room of 155·57 lbs. when the weight is 100 lbs.

242. To show that the resultant tension is due to the

weight of band below the point taken, let *h i*, in Fig. 49, be one-half of a catenary curve *h i y*, the vertex of which is at *i*, where the tension must be purely horizontal, because its direction at that point is perpendicular, to the vertical middle line *z i*, and is therefore horizontal; and as the line of direction *x i t* forms a tangent to the curve at *i* in the same way as *h r* forms a tangent to the curve at *h*, it follows that as the horizontal tension is the same at all points of a full curve, and is therefore equal at all points to the proportionate horizontal line *t r*, the tangential tension must disappear in the horizontal tension without increasing it, when the tangent to the curve at *i*, and the horizontal line *t r* coincide; just in the same way

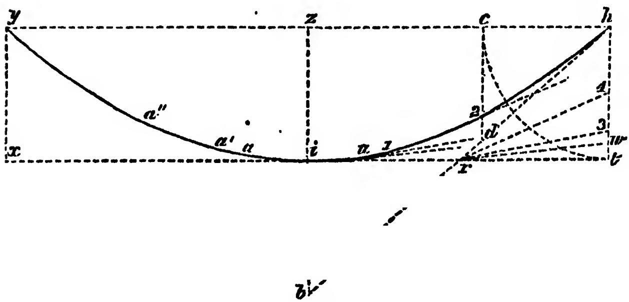


Fig. 49.

as the tangential tension would disappear and leave the 100 lbs. simple weight of the band alone in tension, were the tangent to the curve, represented by *h r*, to coincide with *h t*; because this would show that no horizontal force existed to push the force *v h*, Fig. 47, out of its vertical direction.

243. When the lower pulley is at the vertex or lowest point *i*, it has simply to bear the horizontal tension that would be borne at that point by the other half *i y* of the full curve, Fig. 49, the vertical tension being all borne at *h*.

244. When the lower pulley is, say, at *u*, so as to give the tangential line *r w* transferred from Fig. 46, we have *r w* plainly in excess of *r t*; and, as the force in *r w* is the resultant of the two forces *w t* and *r t*, as explained in reference to Fig. 47, it follows that as *r t* for a full curve represents a quantity constant for all points between the upper and the lower pulleys (the practical pound weight value of this quantity being ruled, however, by the weight of the tension and the angle of suspension at *h*), the excess in *r w* is due to the fractional vertical weight *w t*; consequently, *w t* must be acting at the point of tension on the lower pulley, and must represent the weight of band required at *u* to complete the curve to the vertex *i*, because *h t* represents the weight of the half-curve *h i*, and *h w* the weight from *h* to *u*.

245. It must be borne in mind that, with a given weight, all difference as regards tension is referable to difference of angle, and therefore to the direction of the pulling force in relation to the direction in which weight naturally gravitates.

The simple weight remains the same through all the varying tensions, just as a pound weight is the same at the end of a lever 5 feet long as it is at the end of a lever 1 foot long: but the effect produced by it at the end of the former is 5 times as much as produced by it at the end of the latter.

246. We have been assuming that the distance between the pulleys of Fig. 46 is great, and that the band is placed upon them so loosely that the tension is derived from the deadweight acting freely in the curve.

It is clear, therefore, that with a given distance between the pulleys, the greater the weight of the band, the greater will be the driving power.

247. When at rest, as before observed, the tension in the two lines will be nearly equal, and the degree of stretching will be correspondingly equal.

The whole tension T_1, for the state of rest, must be sufficient to form $T + T'$ for the work that has to be performed, so that, when the stretch is increased on the side T, the contraction on the side T' will take up what T gives (paragraph 197), but this has reference mainly to very flat angles.

Were the band non-elastic, the curve in which T acted would flatten, and that of T' would become more hollow, by reason of the slack thrown over by T. (Paragraph 233.)

248. As shown in Fig. 46, *h i* forms only a fractional part of a half catenary, but the continuation downwards so as to form a half curve, places the lowest point at *n*.

249. Let us here suppose that the whole tension T_1 for the state of rest, is insufficient to give $T + T'$ for the work to be performed, consequently, that the band has to be shortened so as to increase T_1, by flattening the angles at *h* and *j*.

The tangent to this new curve *h a i* lies in the radius *h v*, and *b r* is the sine of the now angle *m h v*.

The fractional curve *h a i* forms part of a half catenary much greater than *h i n*, and as the horizontal and vertical forces *t q* and *h t* are components of the tension *h q*, it follows that as they must increase or decrease with the resultant in their relative proportions, the deadweight *h t* for the resultant *h r* is less than the deadweight for *h q*. Hence, if we represent *h r* by the weight *e* suspended from the lower pulley, and *h q* by the greater weight *f*, and let the lower pulley *c* swing downward, employing the axle *p* of the upper pulley as the centre of this motion till *h q* coincides with *h r*, so as to make the angle *m h v* equal to *m h l*, we have in this new position the greater tension *h q* represented by the weight *g*, which is less than *f* but greater than *e* (paragraph 255). Before we can make the reason of this clear, however, the varying action of the forces at different points in the curve must be explained.

250. The tangent to the curve at *u*, Fig. 49, when transferred to the point *r*, gives the resultant *r w*, and similarly, tangents to the points 1 and 2 give the respective resultants *r* 3 and *r* 4; *w t* represents the vertical tension for the curve *u i* at *u*; and *t* 3 and *t* 4 respectively represent the vertical tensions for the curves 1 *i* and 2 *i*; that is, for the whole weight of the curve below the point that the respective tangents touch.

251. When the lower pulley is placed beyond the vertex *i*—say at *a*—so that the distance *i a* is equal to the distance *i u*, the vertical tension at *a* is equal to *w t*; the horizontal tension to *r t*, and the resultant to *r w*, the same as for the point *u*, the tension at the point *i* being horizontal only.

If *a* be placed higher up, so that *i a′* is equal to *i* 1, then the vertical tension at *a′* is equal to *t* 3, and the resultant to *r* 3; and similarly, when *i a″* is equal to *i* 2, the vertical tension is equal to *t* 4, and the resultant to *r* 4.

When *a* is carried up to *y*, on a level with *h*, the tensions, as might be expected, are there all the same as for *h*.

252. Now, with reference to the weights *e*, *f*, and *g*, we have *e*, equal to 90 lbs. (paragraph 241) represented by *h w*, the tangent to the curve at the lower pulley being parallel with and represented by *r w*.

But, in flattening the curve to *h a i*, Fig. 46, and then sinking the lower pulley till the angle *m h v* coincides with the angle *m h l*, we have the tangent to the curve at *i* no longer parallel to *r w*—but, say, to *r* 3, the tangent to the point 1 of Fig. 49. (Paragraph 259.)

The angle *m h l* of Fig. 46, or its equivalent *h r t* of Fig. 19, is 40°, as before.

The three forces—vertical, horizontal, and resultant, are again represented by *h t*, *t r*, and *h r*, and the horizontal and angular tensions at the lower pulley by *t r* and

: 8; so that, when $h\ t$ represents the vertical weight for the full curve, $h\ 8$ represents the weight for the part-curve $h\ 1$, which we assume to be equal to $h\ a\ i$ of Fig. 46.

253. As before observed (paragraph 249) $h a i$ of the angle $m\ h\ v$ forms part of a curve $h\ i\ n'$ of greater span and depth than $h\ i\ n$ for the angle $m\ h\ l$, and the span and depth will be still greater when the angle $m\ h\ v$ is increased to coincide with $m\ h\ l$—call this greatest curve $h\ i'\ n''$; consequently, as the distance between the pulleys is kept constant, we ought, in Fig. 49, to have $h\ u$ and $h\ 1$ of equal length, but on curves of different dimensions, all starting from the point h, the greater curves $h\ i\ n'$ and $h\ i'\ n''$ having their lowest points lower than i, and the opposite point of suspension farther distant than y from h.

As this, however, would involve a network of lines to represent the respectively different triangular forces $h\ t$, &c., we think it simpler to employ the single curve $h\ i\ y$; the tangent to the curve at the several points i, u, 1, and 2, will determine the respective values of the different dimensions; keeping this always in mind, that the tangent to the curve, as regards direction and angle, is identical with the cosecant of the horizontal angle at h.

254. Though $i\ u$, Fig. 49, be empty of weight, when the lower pulley is at u, the effect upon the resultant tension or the tension in the band at u is the same as would be produced there were the part between h and u removed and u made the upper pulley for the curve $u\ i$; because the deadweight acting vertically at h is as 1 to the cosecant of the horizontal angle at h; and the tension in the band at u is as the cosecant of the different horizontal angle at that point, to the weight 1 of the band below in $u\ i$, when we assume the curve to be complete from h to the vertex or lowest point i, and take u as merely an inter-

mediate point in the band. Consequently, when the lower pulley is at *u*, we have the tension belonging to this point borne by the pulley; and similarly for the other points 1 and 2.

255. Now, as the driving power must be measured by its weakest point, and as it is weakest at the lower pulley, we have the cosecant of the horizontal angle at *u* representing the driving power; and, as the driving power is greater for the flattened curve *h i n'* than for *h i n* of Fig, 46, it is clear that when the angle *m h v* is opened to make it coincide with *m h l*, and is thereby lowered in tensional value by the lessening of the cosecant, we have the deadweight — that is, the vertical component of the greater tension in the flattened curve *h a i*, so in excess of the original weight, that, when we subtract 90 lbs. as the actual weight of the band borne at *h*, we have more than the original remainder of 10 lbs. for the weight of curve below *u* or *i* at the lower pulley, and, consequently, have the working tension represented by the weight *g* for *h i' n''* greater than *e* for *h i n*, but less than *f* for *h i n'*. (Paragraph 257.)

256. By measuring the lines *r w*, &c., of the diagram, we get the forces proportionately, and give a positive value to them, when we know the force of any one of these lines—say *h w* for deadweight of band between *h* and *u*, Fig. 49, — *h t* being for the full half curve *h i*, of that figure.

In paragraph 241 we reduced the deadweight from 100 to 90 lbs—that is, $\frac{1}{10}$th—and now wish to know the horizontal angle *w r t*; so that we may get the tabular value of the cosecant *r w*.

The tabular value of the horizontal tension *r t* being the same for all points of the curve, we employ it here as a standard; and as the deadweight *w t* is as 1 to 10 for *h t*, it is clear that for *r t*, for a weight equal 1 in *u i* to be of

equal value to a weight equal 10 in $h\,i$, the ratio of $r\,t$ to $w\,t$ must be as 10 to 1. Hence,—

$$\begin{array}{l} \quad\ \text{Tabular value.}\quad \text{lbs.}\quad \text{Weight.} \\ rt = 1{\cdot}1917 \times 90 = 107{\cdot}253 \text{ tension.} \\ \ ,, \ = 1{\cdot}1917 \times \ \ 9 = 10{\cdot}7253 \text{ ratio.} \end{array}$$

$w\,t = \overset{\text{lbs.}}{10} \times 10{\cdot}7253 = 107{\cdot}253 = r\,t =$ horizontal tension at u.

Further, we find that 10·7253 is the value of the cotangent $r\,t$ for 5° 20′ nearly, and that the cosecant of this angle is 10·76 nearly; so that—

$r\,w = \overset{\text{lbs.}}{10} \times 10{\cdot}76 = 107{\cdot}6$ lbs. = working tension on the lower pulley at u.

As before determined, paragraph 241, the tension at h in connection with these quantities is

$$h\,r = \overset{\text{lbs.}}{90} \times 1{\cdot}5557 = 140{\cdot}130 \text{ lbs.}$$

257. Now, as the angular tension at h in $h\,r$ for $h\,u$ has 90 lbs., $h\,w$, for deadweight, and as we have this weight of band constant (disregarding the small difference due to difference between the length of the flatter and the more hollow curves), we have the resultant tension at h in $h\,q$ for $h\,l$, Fig. 46, also due to 90 lbs. weight, so that, as the greater tension $h\,q$ represents greater weight in the curve, the number of times $h\,r$ is contained in $h\,q$ gives the number of times the weight of the curve $h\,i\,n$ for $h\,r$ is contained in the weight of the curve $h\,i\,n'$ for $h\,q$: thus, assuming that the angle $m\,h\,q$ is 35°, the cosecant of which is 1·7434, we have

$$\frac{1{\cdot}7434}{1{\cdot}5557} = 1{\cdot}120 \text{ ratio;}$$

consequently, when the weight $h\,t$ for $h\,i\,n$ = 100 lbs., the weight of $h\,i\,n'$ = 112 lbs.

258. Further, as $t\,q$ is constant for all points from i to h, and as it is for 35° equal to

$$\overset{\text{cotangent.}}{1{\cdot}4281} \times \overset{\text{lbs.}}{90} = \overset{\text{lbs.}}{128{\cdot}529} \text{ horizontal tension at } h,$$

for the 90 lbs. actual weight of band suspended from that point, it must be of equal amount at the point 1 for the weight that in effect belongs to 1 i; therefore

$$\overset{\text{lbs.}}{112} - \overset{\text{lbs.}}{90} = 22$$ lbs. weight for 1 i, represented in the resultant tension r 3; and

$\frac{90}{22} = 4{\cdot}09$ ratio, by which to multiply the tabular value of the cotangent for the angle of 85° at h to get in the product its tabular value for the flatter horizontal angle at 1.

For convenience, we employ the single curve and single triangular figure $h\,r\,t$ for the several curves under treatment; hence, in the case of the tension due to the angle of 85°, $h\,q$ is represented by $h\,r$ and $t\,q$ by $t\,r$, and consequently the horizontal angle at 1 by $t\,r$ 3. The different tabular values employed for the different angles named, give the results sought for,

$$\overset{\text{Cotangent.}}{1{\cdot}4281} \times \overset{\text{Ratio.}}{4{\cdot}09} = 5{\cdot}84 \times \overset{\text{lbs.}}{22} = 128{\cdot}5 \text{ lbs.}$$

And, as 5·84 is the cotangent of 9°43′ nearly, we have the cosecant r 3 of this angle = 5·925, so that

$$\overset{\text{Cosecant.}}{5{\cdot}925} \times \overset{\text{lbs.}}{22} = 130{\cdot}35$$ lbs. resultant tension in r 3.

Thus, in flattening the curve from $h\,i\,n$ to $h\,i\,n'$, we have increased the working tension on the lower pulley from 107·6 lbs. to 130·35 lbs., and

$$130{\cdot}35 - 107{\cdot}6 = 22{\cdot}75 \text{ lbs. gain in power.}$$

259. In lowering $h\,q$ to coincide with $h\,r$, we reduce the tabular value to equality with $h\,r$; but as the curve between the pulleys is now flatter than for the original catenary $h\,i\,n$, we have the resultant tension at 1 approximating nearer to the resultant tension at h; that is, the cosecants for the horizontal angles at these two points are nearer equality, the weight of the arc 1 i being now represented by t 3, and the weight of h 1 by h 3; consequently, the working tension represented by the weight

g on the lower pulley for $h\,i'\,n''$ is greater than that for $h\,i\,n$, represented by the weight e; but it must be less than for $h\,i\,n'$, represented by the weight f, as we can at once make plain by swinging the lower pulley downward from p as a centre till the two pulleys are in vertical line. It is evident that the band in this position will be quite loose on the lower pulley, because there is no longer a curve to take up the full length.

We may here observe, however, that it is only where power has to be transmitted between two points at considerable distance that the values of the tensions in the catenary curve need be taken into consideration; because in all ordinary cases of inclined or horizontal belting the distances are so short, and the curves so flat, that they may be regarded as practically straight.

SECTION XVIII.

260. We have employed different terms in reference to the diagonal lines, $h\,r$, $h\,q$, $r\,w$, &c., calling them co-secants, secants, resultants, and tangents, and endeavoured to make the references plain when the terms were changed, but think some circumstantial explanation is here necessary before proceeding further.

261. A tangent to a curve is no more than a line of direction which touches the curve on the outside, and is at right angles to a radial line between the circle centre and the point of contact; and, being of no definite length, is of no value until the angle which it forms with a horizontal or a vertical line is known, so that the value is ruled altogether by the angle.

Thus, the lines $c\,d$ and $e\,f$, Fig. 41, are tangential to the pulley rims; but, as the lines $m\,p$ and $t\,r$ of Fig. 43 are also tangents to the circle at the respective points

m and *t*, it is easy to see that the term tangent means no more than any line from the rim at right angles to the radius, until we employ it in relation to an angle, when it at once assumes a tabular value.

Let us take the horizontal angle *m h l*, Fig. 46; *h l* is the tangent alike to the rope curve and the pulley circle at *h*, and as such, it gives the direction merely of the force, and no more; that is, it simply fixes the number of degrees in the angle that it forms with the horizontal line *h m* or the vertical line *h t*, and is quite distinct from the tabular tangents *m p* and *t r*.

These latter tangents *m p* and *t r* clearly express the power of the angles, and, therefore, have a place in the tables, which the tangent to the curve only has not, until its length is determined at the points crossed by *m p* and *t r*, and then it ceases to be termed a tangent, as it has become the cosecant and secant of the respective angles *m h p* and *t h r*, Fig. 48.

The length *h l* in the diagram is equal to the radius of the circle, but we have here employed it for convenience to express direction merely.

262. The resultant has reference to the resolution of forces in Fig. 47, and the cosecant or secant to the tabular value of the resultant with reference to the angles which it forms with the vertical and horizontal lines.

When determining the forces in the lines of Fig. 46 and 48, we employed the sine and cosecant only of the horizontal angle *m h l*; whereas, when we are dealing with the vertical angle *l h t*, we have to employ the cosine and secant of *l h t* to get the same results.

263. A brief explanation of the values of the whole of the lines in Fig. 48 may sufficiently show the meaning of the prefix "co," and the reason for employing the values with the prefix and without.

The angle *m h t* must necessarily be a right angle in order to get the full values of the sines and cosines, &c.

A tangent, as before observed, must always be at right angles to the radius of the curve or circle it proceeds from; so that the tangent *m p* is at right angles to *m h*, and is, therefore, parallel to the sine *o l* and the radius *h t*.

The line *r t* is a tangent to the circle at *t*, but as it faces the angle *l h t*, formed with the vertical line *h t*, and belongs, therefore, when the angle *m h l* is employed, to what is termed the complementary angle, it is called the co-tangent to the angle *m h l*.

The line *h p* is the secant, and *h r* the cosecant of the angle *m h l*, and *o l* equal to *h x* is the sine of that angle; the cosine is *o h* or *l x*, these two being equal; the versed sine is *m o*, and the co-versed sine *x t*.

264. A complementary angle is simply its difference from a right angle 90°; so that as we made the angle *m h l* equal to 40°, the angle *l h t* is equal to the difference, viz. 50°, and is complementary to the first, that is, it simply completes the full complement of 90°.

In referring to Tables of Sines, &c., it is simply necessary to know this, when we require the values of any of the lines belonging to the triangle that contains this complementary difference, because all these lines are distinguished by the prefix "co."

When we employ the angle *l h t*, the angle *m h l* becomes the complementary difference, and the prefix "co" is transferred to its lines; so that now *r t* = tangent, *h r* = secant, *l x* = sine, *h p* = cosecant, *m p* = cotangent, *o l* = cosine, *x t* = versed sine, and *m o* = co-versed sine.

Thus the horizontal tension *r t* is the cotangent of the angle *m h l*, and is likewise the tangent of the complementary angle *l h t*.

265. As regards the three forces, vertical, horizontal, and oblique, represented by the three sides of the triangular figure, as explained in reference to Fig. 47, we are not, as before remarked, confined to any given area of

space within the triangle; so that, in actually measuring the forces by the Fig. 47 method, that is, by the proportionate length of the sides merely, we can use the area *h l o*, or *h p m*, Fig. 48, or indeed any other possessed of the same angles; and might use either of the areas shown in the vertical angle of that figure, viz., *x h l* or *t h r*, because, by construction, the three angles therein are the same as in *h p m*, though their positions in the diagram are reversed.

As the vertical lines, however, *o l* or *m p* would have to represent the simple weight of the belt, in the manner we have hitherto made *h t* represent it; and as *o h* or *m h* would have to represent the horizontal tension, the action is not so plain to observation, because the resultant *h l* or *h p* is thereby made the base of the figure, in a manner, perhaps, more consistent with compression than with tension.

266. Moreover, in employing the triangle *h p m* for direct measurement of the sides, we must not think of *h m* as the radius, because, practically, it is taking the place of the cotangent *t r* of the tabular method; and, as the tabular value of the cotangent *t r* for the angle of 40° *m h p* is 1·1917, whereas the radius in the tables is always equal to 1·00 simply, it would be confusing to retain this simple value of 1·00 for *h m*, because then the vertical line that represents the weight, which we have hitherto treated as equal to the radius 1·00, would require to be $\frac{1{\cdot}00}{1{\cdot}1917} =$ ·83 only; so that, as the horizontal and the resultant tensions are simply powers of the weight, this would be making them powers of a fraction, instead of powers of a single whole number.

267. We therefore find it more convenient to use the co-triangle *t h r*, the angle *t r h* of which is equal by construction to the angle *m h l*. The vertical side *h t* represents at once both the radius and the simple weight of the

belt; and as we may assume the weight between the pulleys, and therefore the side *h t*, to be always = 1·0, while the ratio of the other sides *h r* and *r t*, with respect to this 1·0, varies with the angle of suspension, and increases with the tension as the angle *m h l* flattens; whereas the sides *m p* and *h p* decrease as the tension increases, we employ the tabular values of the complementary sides *h r* and *r t* as multipliers of the simple weight, thereby to get the tension; or, as in paragraph 226, we get a like result when we employ the decreasing sine *o l* of the triangle *m h p* as a divisor of the weight.

When the angle of suspension has reference to the vertical line *h t*, that is, when the angle is *t h r*, the figure *h p m* becomes the co-triangle, and as the tension increases with the increase of the vertical angle as the vertex *i* of the belt curve rises, in Fig. 49 we have the secant *h r* of the angle *t h r* increasing in the same degree, and therefore employ it as the multiplier of the weight to get the tension in the belt; the value of the secant *h r* being here the same as when it was termed the cosecant of the horizontal angle.

SECTION XIX.

268. When power has to be transmitted to a distance too great for a single span, the band, most usually a rope, is supported at intervals by bearing pulleys.

Now, the pressure upon these pulleys is found by the same triangular method that we employed to determine the pressures on the pulley axles, in the case of ordinary belting.

269. Thus, in Fig. 50, let *a*, *b*, and *k*, be three intermediate bearing pulleys.

Let the distance *a k* equal 300 feet, and the distance *a b*

equal 150 feet; and let the points *a* and *k* be high enough above the ground between, to allow the angle of suspension at these points to be 40°, with reference to the horizontal

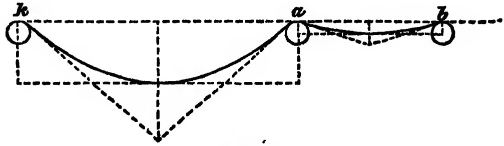


Fig. 50.

line; and let the weight of half the curve between *a* and *k* equal 100 lbs.

We require to find the angle of suspension of the shorter curve *a b* to hold the greater curve *a k* in balance.

270. The angle of 40° is too great for practice; the cosecant is little more than one-and-a-half times as much as the radius which represents the weight of the rope; consequently the power exerted is equal only to one-and-a-half times the weight of rope; a very small power when the work that can be performed is equal only to the difference between T and T′.

We employ this weak angle here merely to allow of wider range in our comparative estimates for different angles.

271. In the case of comparatively flat angles, when the deflexion in the middle is kept constant, and the span lengthened or shortened, the tension alters approximately as the square of the span, so that, supposing the tension to equal 1, and the span 1, the tension will become 4 when we increase the span to 2, the square of 2 being 4; and, when we reduce the span to one-half, or ·5, we have only one-quarter, or ·25 of tension, the square of ·5 being ·25.

272. When we require the tension kept constant, but wish to alter the span, the deflexion will vary as the square of the span nearly; so that, if in a case where the span is

equal to 10, and the deflexion to 1, we increase the former to 20, we must increase the deflexion to 4; thus, $10 \times 10 = 100$, and $20 \times 20 = 400$, and, as the rule operates by the square, we have $\frac{400}{100} = 4$ times the deflexion wanted to keep the tension the same as at first. (Paragraph 279.)

273. The two spans are respectively 300 and 150 feet. We square these lengths, and divide the greater by the less, to find the number of times the tension is increased in the less; thus, $300 \times 300 = 90000$, and $150 \times 150 = 22500$; then $\frac{90000}{22500} = 4$ times.

274. Now, to find the angle of suspension for the shorter span, we proceed thus:—

The vertical line $h\,t$ represents the weight of rope; and, when we make $h\,t$ equal to the deflexion $= 1{\cdot}0$, we have

$$t\,r = r\,i$$
$$z\,i = i\,b;$$

the cotangent $t\,r$ equal to $\cdot 5$ of the half-span $t\,i$, Fig. 49; that is,—

so that $\frac{z\,b}{h\,z}$ = tangent $c\,d$.

Hence the rule for ordinary practice,—

$$\frac{2 \text{ deflexion}}{\frac{1}{2} \text{ span}} = \text{tangent to the angle of suspension.}$$

As $h\,t$ and $t\,i$ for the lesser span $a\,b$ are only about one-half the value of the greater span $a\,k$, while the resultant tensions in the two spans have to be the same, it is clear that for span $a\,b$ the resultant $h\,r$ must be of about twice the tabular value of $h\,r$ for the span $a\,k$; so that, as $h\,r$ for the 40° angle of $a\,k$ is $1{\cdot}5557$, we have—

$1{\cdot}5557 \times 2 = 3{\cdot}1114$, the cosecant $h\,r$ of 18° 45′ angle of suspension in $a\,b$. The cotangent of this angle is $2{\cdot}9459$.

275. Further, as the half-span is equal to twice the cotangent, when the deflexion represented by $h\,t$ for dead-

weight in half-span is equal to 1·0, we have by simple proportion—

2 cotang. : radius 1·0 : : half-span $h z$: radius $z i$ or $h t$

$(2 \times \overset{\text{cotang.}}{2·9459}) = 5·8918$ to 1, ratio of half-span to deflexion when the horizontal angle at h is 18° 45′; so that, as the half-span of $a\ b$ is 75 feet, we have—

$$\frac{75}{5·8918} = 12·73 \text{ feet deflexion of curve.}$$

When the angle is 40°, we have—

$(2 \times \overset{\text{cotang.}}{1·1917}) = 2·3834$ to 1, ratio of half-span to deflexion; and as the half-span in this case for $a\ k$ is 150 feet, we have $\frac{150}{2·3834} = 62·94$ feet deflexion of curve $z\ i$.

276. But, as the rope in the half-span of $a\ b$ must weigh less than in the half-span of $a\ k$, we here require to determine the difference by first finding the lengths of the respective half-curves, and multiplying these lengths by the weight per foot to get the respective weights $h\ t$.

277. The rule here is

$\sqrt{(\frac{1}{2}\text{ span})^2 + \frac{4}{3}\text{ deflexion}^2}$ = length of half-curve; so that

$\sqrt{150^2 + \frac{4}{3}\ 62·94^2} = 166·68$ feet length of half-curve for the greater span $a\ k$; and, as we assumed the weight of this length to be 100 lbs.—

$$\frac{100}{166·68} = ·60 \text{ lb. per foot length.}$$

Again:

$\sqrt{75^2 + \frac{4}{3}\ 12·73^2} = 76·49$ feet, length of half-curve in the smaller space $a\ b$.

Now, $\overset{\text{feet.}}{76·49} \times \overset{\text{lb.}}{·60} = \overset{\text{lbs.}}{45·894}$ weight of half-curve in $a\ b$, and

$\overset{\text{lbs.}}{45·894} \times \overset{\text{cosecant.}}{3·1114} = 142·776$ lbs. tension, which is

155·57 − 142·776 = 12·80 lbs. less than is required to balance $a\ k$.

278. This is owing to the weight of the curve in *a b* for the angle of 18° 45′ being

$$\overset{\text{lbs.}}{50 \cdot 00} - \overset{\text{lbs.}}{45 \cdot 894} = \overset{\text{lbs.}}{4 \cdot 106} \text{ less than the half-weight.}$$

We have consequently either to increase the span of *a b*, to raise the weight to 50 lbs., keeping the angle the same, or to flatten the curve so as to increase the cosecant value, letting the span remain the same.

We will flatten the curve, and find the new angle thus,—

$$\frac{h\,r = 155 \cdot 57 \text{ lbs. tension}}{h\,t = 45 \cdot 894 \text{ lbs. weight}} = 3 \cdot 389 \text{ ratio of cosecant to}$$

radius for *a b*.

3·389 is the cosecant of 17° 10′.

The length of the curve *a b* at this angle will be less than for 18° 45′, consequently the weight will be less; but, as the difference is small, we will not seek to determine it, but will assume the weight to be as found for 18° 45′, and reversing the last expression,

$$\overset{\text{lbs.}}{45 \cdot 894} \times \overset{\text{cosecant.}}{3 \cdot 389} = 155 \cdot 57 \text{ lbs. tension in } a\,b, \text{ balancing}$$

the tension in *a k*.

Were we to adjust to exactness the angle and weight of curve in the given span *a b*, we would have the angle still flatter than 17° 10′ to give the requisite resultant power to the reduced deadweight. (Paragraph 288.)

279. We will here transfer the main angles and lines

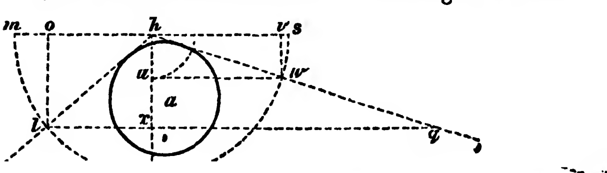


Fig. 51.

from Fig. 48 to Fig, 51, retaining the letters, and making *h* the point where the resultants *r h* and *w h* for

the two unequal curves meet as tangents to the pulley *a*.

We will continue *m h* to *s*; now, by construction, *h m*, *h l*, *h t*, and *h s*, are equal; and, as the sides of the triangles *h x l* and *h o l* are in the same relative proportion to one another as are the sides of the greater triangle *h t r*, we get an equal result by making the radius *h l* act in the room of the cosecant *h r*. And, as *h w* is also a radius, and therefore equal to *h l*, we have *h w* representing the same strain on the rope in the smaller span, as *h l* represents for the greater; because, though the weight of rope in the smaller span is only a little less than one-half the weight in the greater span, we have the ratio of horizontal tension to deadweight of rope in the former a little more than twice the ratio in the latter; so that, assuming the weight in *a b* to be ·5, and the ratio to be 2 to 1, to simplify our estimate, we have ·5 for weight, multiplied by 2 for double ratio, equals 1·0 = the energy of the resultant *h w*; and this is equal to, and balances the weight 1·0 of the greater span *a k*, multiplied by the single ratio 1; so that the energy in *h l* is also = 1·0, the same as in *h w*; and we have simply to divide the 2 for ratio in *a b* by the ·5 for weight, to get the ·4 for tension in *h w*. (Paragraph 272.)

280. We have already remarked, paragraph 265, that the values of the sides of the triangles are fixed by the angles; so that a small triangle possessing the same angles as a larger one, is of equal tabular value with the latter; therefore, the triangle *h u w* is of the same value when *h u* represents the 45·894 lbs. weight, as either of the greater triangles *h x q*, or *h t y*, in which respectively we make *h x* or *h t* represent that reduced weight.

281. In Fig. 51, however, with the resultants *h l* and *h w* of equal length, *h u* and *h x* do not represent the deflexion, as in paragraph 274; so that the cotangents *u u*

and $x\,l$ cannot be measured as respectively representing one-fourth of the whole span.

Fig. 51 is of use, therefore, merely in connection with the tabular values of the angles, and for direct measurement of the angular forces. Thus, the deadweight $h\,u$ for the smaller curve is shown relatively proportionate to $h\,x$ for the greater curve, and the other sides are consequently also in relative proportion, so as to be measurable by one common scale of units.

When the vertical deadweight line is made equal to the deflexion, as in Fig. 50, the lines of triangular force in the smaller curve must be proportioned to a different scale of units to that employed for the larger curve; because, while the deflexion of $a\,b$ is 12·73 feet, represented by $h\,u = 45{\cdot}89$ lbs., the deflexion of $a\,k$ is 62·94 feet, represented by $h\,x = 100$ lbs.: so that $62{\cdot}94 \div 12{\cdot}73 = 4{\cdot}94$ for $h\,x$, to 1 for $h\,u$, in Fig. 50; whereas, in Fig. 51, they are shown in true proportion: $100 \div 45{\cdot}89 = 2{\cdot}18$ for $h\,x$, to 1 for $h\,u$.

282. We will now reduce $a\,b$ to 75 feet span $= 37{\cdot}5$ feet half-span, and find the weight and angle for that altered proportion.

The ratio of the spans $a\,k$ and $a\,b$ being here as 4 to 1, we have

$1{\cdot}5557 \times 4 = 6{\cdot}2228$, equal to the cosecant of $9^\circ\,14'$, the cotangent being 6·1515.

Now $(6{\cdot}1515 \times 2) = 12{\cdot}303$ to 1, ratio of half-span to deflexion; and

$$\frac{37{\cdot}50 \text{ feet}}{12{\cdot}303} = 3{\cdot}048 \text{ feet deflexion.}$$

$\sqrt{37{\cdot}5^2 + \frac{4}{3}\,3{\cdot}048^2} = 37{\cdot}665$ feet length of half-curve of $a\,b$; and

$$37{\cdot}665 \text{ feet} \times {\cdot}60 \text{ lb.} = 22{\cdot}6 \text{ lbs. weight} = h\,u.$$

But, as this weight of half-curve is less than 1 to 4 for the weight of $a\,k$, we will at once as before find the

flatter angle that will enable this light weight to balance $a\,k$, keeping the half-span still 37·5 feet.

$$\frac{155 \cdot 57 \text{ lbs. tension}}{22 \cdot 6 \text{ lbs. weight}} = 6 \cdot 8836 = \text{cosecant of } 8^\circ 21'.$$

The weight of this flatter curve is so little different to the weight for 9° 14′, that we will consider it unaltered; and, as before, reversing the last question—

$$22 \cdot 6 \text{ lbs.} \times \overset{\text{Cosecant}}{6 \cdot 8836} = 155 \cdot 57 \text{ lbs. tension.}$$

283. Now to show that the rule that has the tension and deflexion varying as the square of the span is only approximately correct, but that, as the angle flattens, the approximation becomes closer, we will put the three cases we have just treated into comparative form; thus—

	Span.	Ratio, square.		Deflexion.	Ratio deflexion.
40°	300 ft.	1	×	63 ft.	= 4·93 to 1.
17° 10′	150 ,,	4	×	12·73 ,,	
8° 21′	75 ,,	16	×	3·048,,	= 4·17 to 1.

SECTION XX.

284. As regards the vertical pressure on the axle of the pulley a, we have half the whole weight of each of the two spans; that is, 100 lbs. + 45·894 lbs. = 145·894 lbs. for the 300 and 150 feet spans.

The 155·57 lbs. tension in the rope in the direction of the lines $h\,w$ and $h\,l$ is the resultant of the triangular force, and as such, includes the weight of the material, because without that weight the tension could not exist.

285. The tension which is here exerted in two opposite directions tangentially to the pulley, is increased or diminished with the angle; but the deadweight pressure upon the axle remains constantly equal to the deadweight of the rope; and, according to the angles formed

on the two sides of the pulley, will the line of pressure lie nearer to or further from the vertical direction.

286. We will employ Fig. 52 in finding the line of pressure, and will transfer to it the required lines and angles from Fig. 51.

As *h l* and *h w* are tangents to the pulley circle at the points *b* and *c*, it follows that perpendiculars to these tangents drawn inward from *b* and *c* will meet at the centre of the axle *a*; and it also follows that a line drawn from where the tangents meet at *h* to the centre *a*, gives

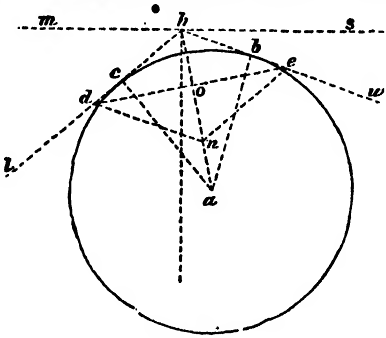


Fig. 52.

the line of pressure belonging to the forces *h w* and *h l*; because the angle *a h b* is equal to the angle *a h c*, and the sides *h c* and *h b* are also equal, and so are the sides *c a* and *a b*, the side *h a* being common to both.

The whole triangles being, therefore, equal, the forces that they represent are balanced on their common base-line *h a*.

But *h a* is not the resultant of the tangential forces. To find it, make *h e* and *h d* equal, then draw *e n* parallel to *h d*, and *n d* parallel to *h e*; the line *h n* is the resultant or measure of the pressure on the axle when *h e* and *h d* represent the 155·57 lbs. tension.

287. We may find the value of the pressure by em-

ploying the angles only; thus, the two right angles *s h t* and *m h t*, Fig. 51, together make $90° + 90° = 180°$, from which we subtract the angles *s h w* and *m h l* to get the angle *w h l* for the 300 and 150 feet spans; so that $17° 10' + 40° = 57° 10'$, and $180° - 57° 10' = 122° 50' =$ *w h l*, which we now divide by 2 to get the equal angles *e h a* and *d h a* $= 61° 25'$ of Fig. 52.

We draw the line *e d*, cutting *h n* at *o*, *h o* being equal to *o n*.

Now, we have already shown that by construction *h n* is the resultant deadweight pressure of the equal tensions in the two spans upon the pulley *a*, and will now prove it by figures.

Let *h o* be the radius, with *h* for centre representing one-half of the 145·894 lbs. weight (paragraph 284), that is, 72·947 lbs. and let *o e* be the tangent and *h e* the secant of the angle *o h e*, which is a vertical angle.

288. The secant of $61° 25'$ is $2·090 =$ *h e* or *h d*, and, as we use this tabular value as a multiplier of the deadweight to get the tension, the sine being used as a divisor, it follows that 72·947 lbs. × 2·090 secant is equal to 152·459 lbs. tension in *h e* or *h d*.

This tension, however, is $155·57 - 152·45 = 3·12$ lbs. less than the true tension. The reason of this is that, as observed in paragraph 278, the angle $17° 10'$ is too great for the exact adjustment of angle and deadweight of the flattened curve in the given span. Reducing $17° 10'$ to the true angle would increase the $61° 25'$ of *o h e*, and consequently would increase the multiplier, that is, the secant, 2·09, so as to give 155·57 lbs. tension.

We do not here seek a closer approximation, because that requires merely a repetition of the process.

72·947 lbs. × 2·132, the secant of about 62°, gives 155·57 lbs. tension: but as in flat curves the deadweight lessens at a slow rate, while the power of the secant increases at a quickly accelerated rate, the true angle will

be a very little more than 62°, and the true deadweight a trifle less than 72·947 lbs.

As the half-weight represented by $h\,o$ gives the tension $h\,w$, when multiplied by the secant $h\,e$, it follows that the tensions $h\,l$ and $h\,w$ divided by the same secant, the angles in the two sides being equal, gives the total pressure on the axle; which, however, as before observed, is simply equal to the deadweight of rope in the two half-spans.

289. Treating the angle $n\,h\,e = 62°$ differently, so as to employ the cosine as multiplier of the tension to get the axle pressure, make h the centre, and $h\,e$ the radius, in this case representing 155·57 lbs. tension, then $o\,h$ is the cosine, and $o\,e$ the sine, so that as ·46947 is the cosine of 62°, or the sine of the complementary horizontal angle of 28°, we have

$$
\begin{array}{rl}
155{\cdot}57 \times {\cdot}46947 = 73{\cdot}0254 & \text{lbs. pressure} = h\,o. \\
2 & \\
\hline
146{\cdot}0508 & \quad ,, \qquad = h\,n.
\end{array}
$$

290. From the resultant tension of the pulling side, represented by the line $h\,r$, Fig. 48, we subtract the weaker resultant belonging to the return side, then multiply the remainder by the speed to get the power in motion of the band. This question, however, has reference not to the intermediate bearing pulleys, but to the pulleys at the ends; and, as the resultant tension of the pulling line is equal to T, the other quantities T′ and Q are found b the rules already given.

291. The pressure on the axles of the intermediate pulleys is, as we have shown, equal simply to the weight of the rope added to the pulley weight, and is not affected by the angles.

But the pulley or drum at the end of the line has, in addition to its own simple weight, to bear the pressure of the whole tensions represented by the resultants, which

act upon the axle in the manner explained in reference to Figs. 11 and 12.

As the power there, however, is exerted in a horizontal direction, whereas here it is exerted at an angle downward,

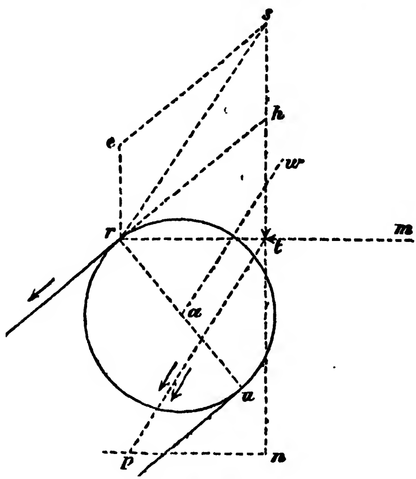


Fig. 53.

the method of finding the line of pressure is better represented by Fig. 53.

292. Let $h\,r$ equal the 155·57 lbs. tension on the rope, exerted in the direction of the arrow.

From r draw the horizontal line $r\,t$, and from h draw the vertical line $h\,t$, and continue it to s; we have here

$$h\,r = \text{cosecant} = \text{pulling force,}$$
$$r\,t = \text{cotangent} = \text{horizontal force,}$$
$$h\,t = \text{radius} = \text{dead-weight,}$$

for the horizontal angle of suspension at h.

Then, employing the same unit of weight, make $h\,s$ equal to the weight of the pulley, and draw $r\,e$ equal and parallel to $h\,s$, and $e\,s$ equal and parallel to $r\,h$.

We have now deadweight acting vertically equal to $t\,s$,

and pulling tension equal to *e s*; the resultant from these two forces is equal to *s r* in direction and measurement, so that *a w*, drawn from the centre *a* parallel to *r s*, is the line of pressure upon the axle.

To prove this, let *t m* be the horizontal component of the tension, and *s t* the vertical; then, as explained concerning Fig. 47, *t p* is the resultant, equal to *s r*.

But the return tension has here to be considered; we may find a second resultant for it alone, but as that resultant will coincide, as regards direction, more or less nearly with *h r*, according to the elasticity of the band, the flatness of the curve, and the ratio of Q to T, it will be found near enough for practical purposes to make *h t* equal to the combined weights of the two tensions at *r* and *u*, the other sides being proportionately increased in value, and *h s*, the weight of the pulley, proportioned to the altered unit; *s r* or *w a* will then be closely approximate to the true line of resultant axle pressure.

293. We may here observe that the tension at the end of the line where the power is applied is greater than the tension at the end to which it is transmitted, the difference being caused by a fractional quantity absorbed at each of the successive intermediate pulleys in overcoming the axle friction and the bending resistance.

294. When the intermediate pulleys are all of the same size we find the loss of power by axle friction for one, and multiply by the number of pulleys. We might assume the whole weight to be upon one, and get the same result theoretically only, because this aggregate load upon one spindle might carry the friction to or beyond the point of abrasion, where the ordinary coefficient of friction would no longer apply in practice.

Our explanation of Fig 13 gives the method of determining the power required on the rim to overcome the resistance on the axle.

295. When the line requires to bend round corners the

resultant of pressure on the axle may be found by the method explained in reference to Fig. 52.

The pressure in this case, however, is less simple than when the line is straight, because, as the curve naturally droops in a vertical direction, its weight at the outer point of the corner pulley, which lies more or less inclined sidewise, acts vertically, so as to put a cross strain upon the pulley.

It would take much explanation to exhibit with precision the contrary forces here at work; we will therefore here simply say that it is advisable to have the spans at elbows in the line as short as possible; so as to get a light load and a flat curve, that the direction of the curve tension in the rope may coincide nearly with the horizontal angular pull caused by the corner; and it is only when this is done that the simple form of Fig. 52 applies.

When the corner spans are great, a side twist is given to the pulley that materially increases the friction at the axle; because, though, when we neglect the weight of the pulley, the axle resistance due to force applied fairly in line with the pulley requires the same power at the rim when working horizontally as when working vertically, the deadweight of the tension of great curves, when the pulley lies inclined from the vertical, acts with leverage represented by the sine *c d* or *g i* of the angle *c g d*, Fig. 58.

With the pulley vertical, we have the deadweight acting with simple pressure without leverage in the line *f g*; and have the axle resistance and the power on the rim required to overcome it, in the ratio of their respective radii, paragraph 77; whereas, with the pulley working in horizontal position in the line *e g*, we have the deadweight acting with whole leverage represented by the radius $g\ e = 1{\cdot}0$, and have the ratio of the leverage as *g e* to *a b*, as regards simple pressure. It is clear,

therefore, that the friction on the journals *a* and *b*, with the load at *e*, must be as many times greater than with the load at *f*, as *e g* is greater than *a b*.

SECTION XXI.

296. Hemp rope stretches more than wire rope under strain, and retains more of the stretch in permanent form; so that the deflexion of the return line curve is increased, and the grip upon the pulley consequently lessened, when the return line is the lower one; the defective elasticity preventing the contraction of the slack line from keeping upon equal terms with the stretching of the working line.

297. When the return line is the upper one, the extra slack thrown over increases the arc of contact and the weight; but this advantage is counteracted by the weakening of the angle.

The permanent stretching in excess of the contraction is progressive, and, as a very small addition to the length of the curve makes a considerable difference in the deflexion, there may soon be a new splice needed, unless special means are employed to take up the extra slack as it is formed.

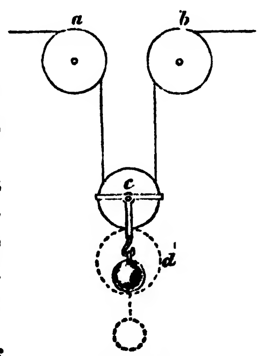


Fig. 54.

298. In the case of short lines, jockey pulleys are sometimes employed; but when these ride on the bottom of the open curve, they are of limited application.

Fig. 54 shows one mode of employing them when the rope is flexible enough to work easily round pulleys of mo-

derate diameter; and, as the permanent stretching in this case mainly affects the curves on the return line, we may place the jockey *c* on this line, and load it so as just to balance the required tension there.

299. When placed on the pulling line, as it often is, it is clear that its force must be equal to T, whereas on the return it need be equal only to T′.

When slack from the pulling side is produced by the permanent stretching of the band, the resultant tension of the return curve is reduced to less than the jockey there was loaded to balance; so that the weight of the jockey being now greater than the weight of tension, the jockey descends say from *c* to *d*, thus taking the extra slack with it, and restoring the tension to its original amount.

300. Fig. 55 shows the jockey acting as in Fig. 54,

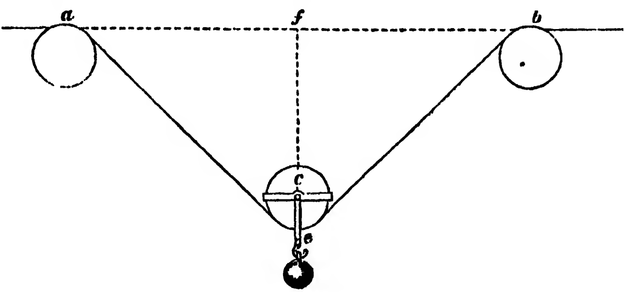


Fig. 55.

but with less advantage, owing to the bearing pulleys *a* and *b* being further apart.

We will suppose the distance *a b* to be so short, and the rope to be so light in comparison with the load *c*, and so flexible that the lines *b e* and *a e* are practically straight, otherwise we would require to treat the case similarly to the curves *h i n*, &c., of Fig. 46.

When the angle *f b e* is known, we can at once find the tension at *b*, by dividing the weight of the jockey

c + weight of rope $b\ e\ a$, by twice the sine of the angle.

We employ twice the sine here because one half of the load belongs to the angle $f\ b\ e$, and the other half to $f\ a\ e$. Before, when using the sine singly as our divisor, we employed only one-half the whole weight of the curve.

The length $b\ e$, when straight, is equal to the square root of the half-span $b\ f$ squared and added to the deflexion $f\ e$ squared, thus expressed, $\sqrt{b f^2 + f e^2} = b\ e$; and also, putting the same values into another form, $b\ e^2$, is equal to $b f^2 + f e^2$, so that the distance which the jockey will descend when increased slack lengthens the line $b\ e$ may be readily found.

301. In Fig. 54, we will suppose that 1 foot of hemp rope slack has to be taken up; this will simply let c descend 6 inches, whatever be the length from $a\ b$ to c at starting.

In Fig. 55, we will suppose the half-span to equal 12·5 feet, and $b\ e$ and $a\ e$ to equal 15 feet each, and that the slack to be taken up is 1 foot; this will give 6 inches to each side, making $b\ e$ equal 15·5 feet, but c has to descend more than 6 inches.

As we have given only the lengths $b\ f$ and $b\ e$, we require to find the deflexion $f\ e$, thus,

$$b\ e^2 = 15^2 = 225$$
$$b\ f^2 = 12{\cdot}5^2 = \underline{156{\cdot}25}$$

68·75, the square root of which is 8·29 feet for the deflexion $f\ e$, before the slack comes over.

When the slack is introduced, $b\ e$ becomes 15·5 feet, which is the square root of 240·25, from which we subtract as before 156·25 for $b\ f$, and get 84 for $f\ e$, the square root of which is 9·16 feet, showing that c has had to descend ·85 foot to take up ·5 foot of increase in the length $b\ e$.

When the angles are known, we can find the descent

of the jockey *e*, by making the half-span *b f*, with *b* as centre, represent the radius, equal to the tabular value 1·0 (the half-span being here a constant quantity), the deflexion *f e* becomes then the tangent, and the rope line *b e* the secant of the angle. So that, multiplying the number of feet in *b f* by the tabular value of the secant, gives us the length in feet of *b e*; and similarly, multiplying the feet in *b f* by the tabular value of the tangent gives us the length in feet of *b e*.

302. In Fig. 56, we will first assume that the two pulleys *a* and *b* are on a level with the endless band of

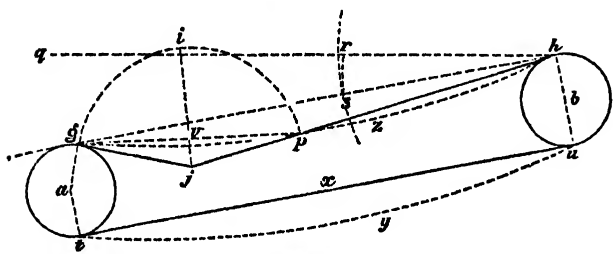


Fig. 56.

the curves *g z h* and *t y u*; and that the jockey, for some reason often arising in practice, can be applied only at a point much nearer to *g* than to *h*.

We require to know the tension when the jockey pressure forms the angle *g j h*.

The rules are thus expressed:

Call the weight of the jockey pulley W, and as this weight is here borne unequally at *g* and *h*, we will call the fractional burden at each of these points *w*. Then

$$\frac{w + \text{weight of band } g\,j}{\text{sine of angle } h\,g\,j} = \text{tension at } g;$$

and similarly,

$$\frac{w + \text{weight of band } h\,j}{\text{sine of angle } g\,h\,j} = \text{tension at } h.$$

303. The weight is borne more heavily at g than at h, yet, the tension at these points is equal, because the sine at g is as much greater than the sine at h as the burden of deadweight is greater, so that, the sine being used as a divisor, we have w the greater, and w the less, producing equal effect at their respective points of suspension.

304. When the tension at the point j is wanted, exclude the weight of the cord.

305. When the required tension is given, and W, the whole weight of the jockey is wanted, then

W = tension × (sine of $h\,g\,j$ + sine of $g\,h\,j$) — weight of band.

Let the angle $g\,h\,j = 5°$, and the angle $h\,g\,j = 20°$, with sines respectively,

$$\begin{array}{l} \cdot 08715 \\ .34202 \quad \text{lbs.} \\ \hline \cdot 42917 \times 100 = 42{\cdot}917 \text{ lbs.} = \text{W}. \end{array}$$

306. Now, to get the relative proportions of the unequal burdens $w\,w$, borne respectively at the two points of suspension, find the ratio of the sines thus:

$\frac{\cdot 34202}{\cdot 08715} = 3{\cdot}92$ parts borne by g to 1 part by h, so that $3{\cdot}92 + 1{\cdot}0 = 4{\cdot}92$, and

$\frac{42{\cdot}917 \text{ lbs.}}{4{\cdot}92} = 8{\cdot}72$ lbs. at h, and consequently 34·197 lbs. at g.

Then, neglecting the weight of the band,

$$\frac{8{\cdot}72 \text{ lbs.}}{\cdot 08715} = 100 \text{ lbs. tension at } h, \text{ and}$$

$$\frac{34{\cdot}197 \text{ lbs.}}{\cdot 34202} = 100 \text{ lbs.} \quad ,, \quad ,, \ g.$$

307. Let us now assume that the reason of the jockey being nearer to g than to h, is on account of difference of level, as shown in Fig. 56.

Let $q\,h$ be a horizontal line, and let the tension in the curve $g\,z\,h$ be equal to 100 lbs. as before. Further, let

the angle $r h s$ be 10°, and the angle $g j h$ 155°14′; then draw the line of pressure $j i$ to divide the angle $g j h$, so that $g j i$ shall be equal to $h j i$, equal to 77° 37′.

Next, from the centre j with any radius describe the arc $g\ i\ p$, and draw the lines $g v$ and $p v$ perpendicular to $j i$.

Now, as the angles at j are balanced, the tabular values for the whole angle $h j g$ are just double the values for the half-angle $h j i$, and as the tension required in the lines $h j$ and $g j$ is 100 lbs. as before, the rule by which we ascertain W, the weight of the jockey pulley, to form the angle $g j h$ is thus expressed:

$$\frac{100 \text{ lbs.} \times 2 \text{ cosine } vj}{\text{cosine } r h} = \text{W},$$

so that as the tabular values are

$$\begin{array}{lll} & & \text{Cosine.} \\ h j i = & 77°37' = & \cdot 21445, \\ r h s = & 10°0' = & \cdot 97629, \end{array}$$

$$\frac{100 \text{ lbs.} \times 2 \times \cdot 21445}{\cdot 97629} = \frac{42 \cdot 890}{\cdot 97629} = 43 \cdot 93 \text{ lbs.}$$

308. W is here shown greater because the angle $r h s$ has lowered the values of the other angles, so that greater weight in the jockey is required to make up the loss when the 100 lbs. tension is wanted in both cases.

309. As bending resistance is inversely as the diameter of the pulley, nearly, the jockey-pulley ought to be as large as possible.

For the given angle of deflexion $g j h$, we will assume different diameters of the pulley riding at j; thus:

Diameter.			Bending Resistance.
1 foot	...	...	·25
0·5 „	...	...	·50
0·25 „	...	...	1·00

The arc of contact for the given deflexion embraces the same number of degrees on the rim of each of these different diameters; but, in the case of the 0·25 foot

diameter, the resistance due to bending to form the given angle *g j h*, is concentrated in an arc of contact that measures in inches only one-fourth the length of the arc for the 1 foot diameter; so that, for the given number of degrees in the whole bend, the bend per inch of length is 4 times as sharp; that is, the bending stress per inch length of the ·25-foot arc is in the ratio of 4 to 1 for the 1-foot arc, paragraphs 210 and 220. It follows from this that the band will wear out soonest on the smaller diameter.

SECTION XXII.

310. When the full value of the tension T of an endless band is wanted, a number of turns is taken round

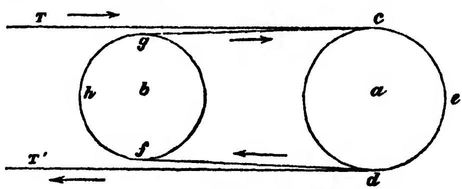


Fig. 57.

either one plain drum or a pair of plain or grooved drums, as in Fig. 57, as explained in paragraph 176.

311. Taking the case of the pair of grooved drums, the friction of the adhesive grip must be greatest in the leading grooves where T is at the maximum, and, as the band slides upon the face of the drum in contracting its length to correspond with T', paragraph 178, the tear and wear in the leading grooves must so exceed that of the succeeding ones for the lesser tensions, that, in course of time, the working diameter of the drum at the former will be so reduced as to take up in the act of pulling a

slightly shorter length of band than the less-worn diameter in the succeeding grooves requires, so that, calling the leading grooves A, and the succeeding grooves B, the band that occupies B must either stretch excessively, or else surge back towards A.

312. Should the band stretch on B, to make up for this deficiency of length we absorb uselessly a portion of the driving power, because the bending resistance and the axle friction are increased by reason of the greater tension due to the stretching.

313. Were each groove in a separate pulley with an axle of its own, as in the tackle-block of Fig. 19, any difference of this sort would be adjusted by the motion of the pulleys, so that B would move as much slower than A as the diameter of A was less than that of B. But, in this case, the driving power would require to be communicated not through the drum, but directly through the band, and, in order to get the minimum tension T′, only one of the two sets of loose pulleys could be fixed in position, because the second set would have to act like the lower sheave-block of a set of tackle-blocks; otherwise, if we fixed both sets, the power at B would require to be equal to the tension at A, neglecting the axle friction and the bending resistance.

314. It is usual to employ a tension-weight acting upon *b*, Fig. 57, to keep the main lines taut.

315. It is clear, however, that a load at *b* must act in the same way as the load *a* on the lower pulley of Fig. 18, because we must here assume the many lines of rope to be acting as one band, nearly, with tension equal to the mean pressure tending to bring the two drums together, paragraph 191.

316. To lessen the weight of the counterbalance, therefore, a third and fourth pulley, with more than one groove in each, are often used; so that the return line does not directly proceed from *d* of the main pulley *a*, but from one

of these additional pulleys, and thus, the weight has to counterbalance merely the tension T′.

The pulley that has the counterbalance weight acts on the same principle as the loaded pulley *c* of Fig. 55; but, according to circumstances, its motion may be horizontal, the plummer blocks it works in either sliding on their beds or moving upon rollers; the counterbalance weight acting vertically at the end of a cord which passes over a small pulley, on a level with the plummer blocks.

SECTION XXIII.

317. The employment of the series of turns round the set of two or more drums saves the rope, by economising the working tension T. We can economise the tension in a much simpler way, but at the expense of the rope, by employing the pulley *a* only, with a single V-groove on

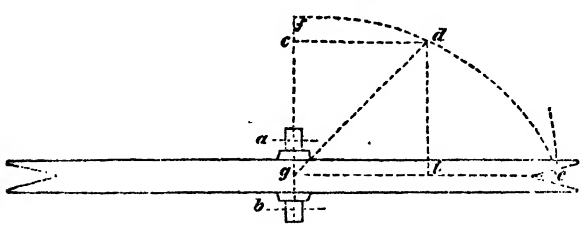


Fig. 58.

the rim, as in Fig. 58, into which the rope is wedged by the pressure; the grip being ruled by the V-angle.

318. Let *a d c* in Fig. 59 be the V-groove, and *r* the rope, and let the pressure required between the jaws be equal to 60 lbs.

From *d* as the centre make the arc *a b c*, then draw *e c*

at right angles to *b d*; the line *e c* forms the sine of the angle *b d c*, and is employed as a multiplier of the whole tension to find the pressure required upon the base *e c* of

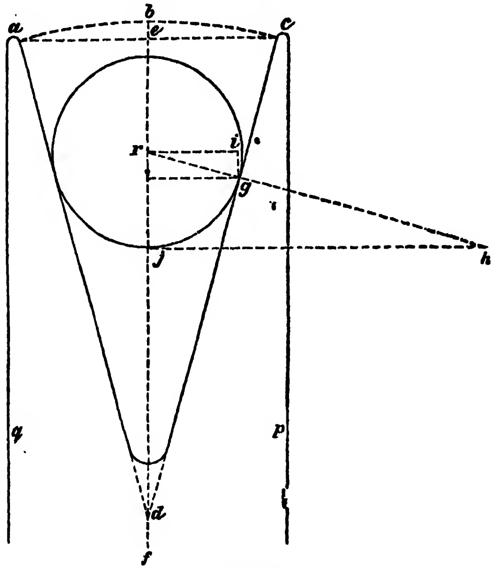


Fig. 59.

the wedge *e d c*, to give a frictional grip upon the side *c d* equal to the tension of 60 lbs.

319. Let us suppose the angle *b d c* to equal 30°, the sine of which is ·500; multiplying this by 60 lbs. gives us 30 lbs. as the pressure required upon the base *e c*, to make the pressure on the side *c d* equal to 60 lbs. acting in a direction perpendicular to *c d*; so that, as *c d* forms a tangent to the circle of the rope *r* at the point *g*, a line drawn from the centre *r*, through the point *g* to *h*, represents the direction of the pressure.

We get the same result when we divide the 60 lbs. tension by the cosecant of the angle: thus, for 30° the

cosecant is 2·00, which gives 30 lbs. pressure required at the base *e c*.

From the lowest point *j* of the circle *r* draw a horizontal line to cut the continuation of the radial line *r g* at *h*, then *r h* is the cosecant of the angle *i r g*, which is by construction equal to the angle *b d c*; and the three sides of the triangle *i r g* are relatively in the same proportion as the sides of the triangle *b d c*.

By construction, also, the angle *r h j* is equal to the angle *b d c*, and the three sides belonging to each are respectively in the same relative proportion.

Thus, in the lesser triangles, *i g* and *r j* correspond with the base *e c*; *r i* and *j h* with *e d*; and *r g* and *r h* with *c d* of the V-groove.

320. We employ the cosecant as a divisor of the combined 60 lbs. tension in the rope, because, as the grip becomes more severe as the angle *b d c* lessens, there is the less pressure needed at the base *e c* of the wedge to produce a given amount of grip, consequently we require the divisor of the tension to increase at a rate corresponding with the increase of the grip.

The secant shortens with the lessening of the angle, until it becomes simply equal to the radius, when the angle flattens to 0°. We therefore employ the cosecant, which increases as the secant lessens, until its power becomes what is termed infinite, at the point where the latter is reduced to simple equality with the radius.

Tension, or angular resultant pressure, however, never reaches this infinite line.

321. Were we to employ the angle *e c d*, which is complementary to *b d c*, and, therefore, equal to 60° when the latter is 30°, we would use the secant, because in this case the thinning of the wedge by the lessening of the angle *b d c* involves the increasing of the angle *e c d*, and, consequently, the increasing of its secant, which is the cosecant of the angle *b d c*.

We find it convenient to employ the angle *b d c* and its cosecant; and will employ only one-half of the whole angle *a d c*, for much the same reason as caused us to treat the catenary curve tension separately, for each side, at an intermediate pulley, when finding the pressure on the pulley, the circle of the rim of which corresponds with the circumference of the rope *r* of Fig. 59; but in Fig. 50 we assumed that the spans were unequal, and, consequently, that the simple weight of rope on one side was greater than in the other; which is equivalent to unequally loading the two half-wedges *b d c* and *b d a* of Fig. 59.

322. These two half-wedges, being of equal angles, will be acting equally, so that when we give to the power of the angle of one the weight belonging to both, we simply get the same as if we had treated each separately with its half-share of the weight.

323. When we employ the whole angle *a d c*, we must either double the load or the length *e d*, when the resistance, and therefore the thrust, acts in the direction *j h*; but when it acts in the direction *r h* we must either double the load or the sloping side *c d*.

324. Were the jaws of the groove *c d* and *a d* not joined together at *d*, that is, were they free to separate by sliding horizontally, the line of resistance would be parallel with *j h*; but being bound together, *d* acts as a centre for the bending leverage of the sides, in which case *r h* is the resistance line.

325. Let us now make *b d c* equal to 15°, the sine of which is ·25881, to be used as a multiplier, and the cosecant 3·8637, to be used as a divisor of the 60 lbs.

Either of these gives us 15·528 lbs. load required at *e c* to produce 60 lbs. pressure on the side *c d*; this is equivalent to 30 lbs. pressure on each of the two sides *a d* and *c d*, the same as found for the angle of 30°; and as this pressure on the sides must be multiplied by the coefficient of friction for iron on iron for wire rope, we have 60 ×

·18 = 10·8 lbs. power in frictional adhesion, produced by 15·528 lbs. load on *e c*, with the angle *b d c* equal to 15°; the same as we get for 30 lbs. in an angle of 30°.

326. The nipping pressure required between the jaws of the groove is equal to 60 lbs. We keep that constant, and merely vary the angle, until the power of the sine and cosecant are such that a weight equal simply to the frictional adhesion of the given pressure is sufficient to produce that given pressure.

327. At the angle of 30° a load pressure of 30 lbs. on *e c* gives a nipping pressure of 60 lbs. on *c d*. Now, *c d* cannot bear this pressure unless there be a resistance equal to it in *b d*, the base of the angle *b d c*; but, as *b d c* is only one-half of the whole angle *a d c*, the base line *b d* merely marks the division of the two halves, so that before the nipping pressure on *c d* can establish itself it has to stretch across to the side *a d*. In reality the whole base *a c* is loaded, so that each side *a d* and *c d* is pressed independently, and has its pressure reacting on the other. We, therefore, employ the action and reaction of one side only for the simpler exhibition of the force.

Were *a d* parallel with *b d*, the pressure transmitted to it from *c d* would have a solid abutment, and the 60 lbs. pressure would, therefore, react upon *c d* whole and entire; but, as it lies at an angle of 30° with *b d*, the transmitted pressure tends to slide outward upon the slope, after the manner of bodies placed upon ordinary inclined planes; so that a transmitted pressure of 60 lbs. would, in the case of the angle *b d a*, meet with a holding resistance on the slope *a d* equal to its own weight, 60 lbs., divided by the sine = ·50, which gives only 30 lbs. resistance on *a d*, and, consequently, a counter-reaction upon *c d* simply equal to this; so that the 30 lbs. load pressure on the base of the wedge produces in the two jaws *a d* and *c d* of the groove a nipping pressure equal to 30 + 30 = 60 lbs.

328. For convenience, we placed the 30 lbs. load pres-

sure on the half-base *e c*, to get the 60 lbs. nipping pressure directly by the angle.

Were we to distribute this 30 lbs. load uniformly over the whole base *a c*, we would thereby have 15 lbs. to each half-base; which would give us 30 lbs. nipping pressure in the groove for each side, seeing that we have either to find for each half-angle *b d c* and *b d a* separately, or, if employing the whole base to represent the load pressure, we have to double the length of the sides of the whole wedge, thereby making the angle for the whole wedge the same as we have for the half-wedge when the load pressure is represented by the length of half the base *e c*.

329. In the action and reaction of the nipping pressure between the two jaws of the groove, we have the forces meeting on the balance in the middle line *b d*, which is, therefore, a neutral line; and it is because of this neutrality that we employ the sine and cosecant of *b d c* in place of those for the whole angle *a d c*.

330. It is easy to see that as regards frictional resistance the pressure of 30 lbs. on each side of the rope is equivalent to a pressure of 60 lbs. on one side only, such as we have when the rope rubs on a plain surface with a tension of 60 lbs.; because we have twice the area of surface in contact, so that 2 square inches, multiplied by a pressure of 30 lbs., gives 60 lbs., the same as 1 square inch multiplied by 60 lbs.

331. To find the angle that will reduce the load or tension pressure on *e c* to an equality with the frictional adhesion, we divide the 10·8 lbs. adhesion by the 60 lbs. pressure and get ·180, which is the sine of 10° 22′ nearly; the cosecant of which is 5·55719, by which we divide the 60 lbs., and get 10·8 lbs. load required on *e c*, to give 60 lbs. nipping pressure on the side *c d*.

332. The sine and the coefficient of friction agree here, because the 10·8 lbs. frictional adhesion is, to the whole pressure, in the same ratio as the coefficient to the whole

number 1·0; or as the sine to the tabular radius 1·0; both sine and coefficient being employed as multipliers of the pressure to get the fractional effect.

The sine is a fractional part of 1·0, and represents the number of times the deadweight, or radius, is contained in the resultant pressure, or cosecant, and the coefficient is similarly a fractional part of 1·0, representing the number of times the frictional adhesion is contained in the pressure, so that, having the coefficient, we may at once use it as the sine, when the load pressure *e c* requires to be simply equal to the frictional adhesion of the nipping pressure between the jaws.

In the case of the catenary curves of Fig. 46, the sine and cosecant were employed to find the tension produced by the angle; whereas we have here used them to find the compression; the powers of the angles applying equally to tensile and compressive strains.

SECTION XXIV.

333. The friction that occurs in a groove of the Fig. 59 description is severe upon both groove and rope, because the rope on leaving has to tear itself away from the grip, and would soon wear through the outer fibres of the strands in so sharp an angle as $10^\circ\ 57' + 10^\circ\ 57' = 21^\circ\ 54'$ for the whole groove *a d c*; the angle *a d c* is, therefore, never so great as this.

The rope has to slide inward or downward in the groove on entering, and outward on leaving; and as force is required to withdraw it from the grip on the leaving side, it is clear that the return tension must supply the releasing force.

This force, however, in amount is altogether dependent upon the angle of the groove, the degree of compressibility in the rope, and the coefficient of friction between the two surfaces.

Sometimes a plough is placed in the groove at the point

where the rope is required to leave, but this does not lessen the frictional wear, nor the force required to effect the release.

In the case of inclined waggon ways. with up and down lines, the empty waggons descending balance an equal weight of the full waggons ascending, so that the grip of the groove need be equal only to the difference.

334. In Fowler's clip pulley the groove is formed of short hinged jaw blocks, represented in cross section in Fig. 60; in which *a* and *b* are the blocks, *c* and *d* the hinges, *e* the rope, and *f* the body of the pulley.

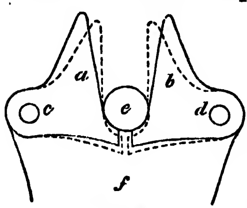


Fig. 60.

The descent, under pressure, of the rope in the groove causes the jaws to close and nip it with intensity, increasing with the pressure or tension; and, as the nip is ruled by the tension, it follows that the compression of the jaws, $30 + 30 = 60$ lbs., is no greater than we have for the same tension of rope between the fixed jaws of Fig. 59.

Very slight force is required at once to free the rope from the grip on the leaving side; because the jaw blocks then open easily on their hinges; the return tension T' has, therefore, less to do than when the jaws are fixed.

335. As regards the working power of ropes, we will here simply give the tables of strengths supplied by R. S. Newall & Co. of Gateshead.

The working and breaking strains given in the two right-hand columns, require that ropes composed of the three different materials named, viz., hemp, iron wire, and steel wire, be respectively of the different circumferences and weight per fathom noted in the left-hand columns to be of equal strength.

Thus, the circumference of hemp rope requires to be greatly in excess of that for iron, and still more in

excess of that for steel; the weight per fathom of the hemp rope is in excess in nearly the same proportion as the circumference.

Wire rope of ordinary form is stiffer than hemp rope of equal strength; but, by modifying the form, the flexibility may be increased so as to allow the wire ropes freely to work round small diameters.

As we before observed, the stretch that occurs in ropes made of hemp renders them less suitable than wire rope for driving bands, even though the coefficient of friction between hemp and iron is about ·50 of the pressure, while the coefficient between iron and iron is only ·18.

336. TABLE I.

Hemp.		Iron.		Steel.		Equivalent strength.	
Inches circumference.	lbs. weight per fathom.	Inches circumference.	lbs. weight per fathom.	Inches circumference.	lbs. weight per fathom.	Cwts. working load.	Tons breaking strain.
2¾	2	1	1	..	..	6	2
,,	,,	1½	1½	1	1	9	3
3¾	4	1⅝	2	..	..	12	4
,,	,,	1¾	2½	1½	1½	15	5
4½	5	1⅞	3	..	..	18	6
,,	,,	2	3½	1⅝	2	21	7
5½	7	2⅛	4	1¾	2½	24	8
,,	,,	2¼	4½	..	..	27	9
6	9	2⅜	5	1⅞	3	30	10
,,	,,	2½	5½	..	..	33	11
6½	10	2⅝	6	2	3½	36	12
,,	,,	2¾	6½	2⅛	4	39	13
7	12	2⅞	7	2¼	4½	42	14
,,	,,	3	7½	..	..	45	15
7½	14	3⅛	8	2⅜	5	48	16
,,	,,	3¼	8½	..	..	51	17
8	16	3⅜	9	2½	5½	54	18
,,	,,	6½	10	2⅝	6	60	20
8½	18	3⅝	11	2¾	6½	66	22
,,	,,	3¾	12	..	..	72	24
9½	22	3⅞	13	3¼	8	78	26
10	26	4	14	..	..	84	28
,,	,,	4¼	15	3⅜	9	90	30
11	30	4⅜	16	..	..	96	32
,,	,,	4½	18	3½	10	108	36
12	34	5[illegible]	20	3¾	12	120	40

Smaller sizes than the least in this first Table are distinguished by numbers in Table II., which, however, does not give the comparative strengths of hemp. The circumferences and the weight per 100 feet are the same for the steel as for the iron, the respective strengths alone being different.

337. TABLE II.

Steel Cord.					Iron Cord.	
No.	Breaking strain.	Working load.	Inches circumference.	lbs. weight per 100 feet.	Breaking strain.	Working load.
	lbs.	lbs.			lbs.	lbs.
1	5040	1344	$1\frac{1}{8}$	16·5	2520	672
$1\frac{1}{4}$	3840	896	$\frac{15}{16}$	9·5	1920	448
$1\frac{1}{2}$	2760	672	$\frac{3}{4}$	7·7	1380	336
$1\frac{3}{4}$	1920	448	$\frac{21}{32}$	6·0	960	224
2	1200	300	$\frac{16}{32}$	3·5	600	150
3	720	180	$\frac{12}{32}$	2·1	360	90
4	500	120	$\frac{9}{32}$	1·5	250	60

SECTION XXV.

338. It sometimes happens that a loaded wheel, propelled by means of cords wound round the rim, is required to travel up and down an inclined or a vertical toothed rack. We will here ascertain how the power of the cord is affected by the angle it works at.

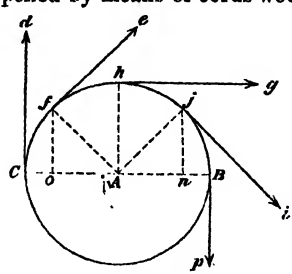


Fig. 61.

In Fig. 61 let A be the centre of a toothed wheel and cord drum of equal diameter, C B; and let B be the point of rest upon the toothed rack, and, therefore, the

fulcrum upon which the leverage of the pulley-load acts.

We will assume that the centre of gravity of the pulley-load is at the centre of the axle A; consequently, the load is acting with a leverage equal to A B, which we will call 1 foot, and will call the load 100 lbs.

339. If we make the cord lead away in the direction C *d*, it is plain that a power of 50 lbs. will be sufficient at *d* to balance 100 lbs. at A, because the leverage C B is twice the leverage A B.

But, if the cord lead away in the direction *h g*, the power required at *g* must be equal to the load A, because the leverage of the power at the point *h* is simply equal to the leverage A B of the load.

If the pull be in the direction *f e*, the power required will then be as much less than the load, as the load-leverage A B is less than the power-leverage *o* B; and if the direction of the pull be *j i*, the power required will be as much greater than the load, as A B is greater than *n* B.

If the cord lead in the downward direction B *p*, it is clear that it can exert no power at all, and that the wheel will simply run down the rack, and wind the cord round its rim as it runs.

340. Fig. 62 represents in simple form the changing value of the power, according to its leverage; the load, and fulcrum, and cord lines being all lettered as in Fig. 61.

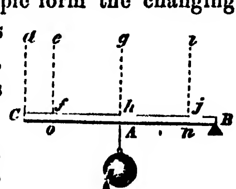


Fig. 62.

From the points *f* and *j*, in Fig. 61, we draw lines perpendicular to C B, and cutting it at the points *o* and *n*; these perpendiculars are sines of the respective angles *f* A C and *j* A B.

The third perpendicular *h* A is radius to the circle,

and is common to the two right angles *h* A C and *h* A B.

Each of these right angles represents a power of 1·0; thus, *h* A B represents a power of 1·0 for the radius A B, and *h* A B + *h* A C represents a power of two for the radii A B + A C = B C; hence, the fractional power of A *o* is added to the power 1·0 for A B, to give the power of *o* B, and the fractional power of *n* B operates simply as the fractional part of the power 1·0 for A B.

As the cord line forms a tangent to the circle at each of the points C, *f*, *h*, and *j*, it is exerting a pull at right angles to the radius at all these points; so that were the fulcrum at A, a given power would act equally at all points of the wheel's circumference; whereas the fulcrum being at B, the perpendiculars from the cord lines must all be drawn to B in place of to A, as we now do in Fig. 63; thus *k* B is at right angles to *f e*, *l* B to *h g*, and *m* B to *j i*.

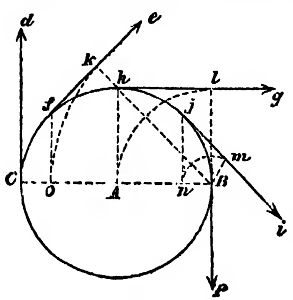


Fig. 63.

341. The length of the load-leverage A B being 1·0, the length of these perpendiculars gives the length of the power-leverage.

To prove this, from B as the centre describe arcs from *k*, and *l*, and *m*, so as to cross the line C B; the points of crossing coincide respectively with the points *o*, and A, and *n*, got in Fig. 61 by simply dropping perpendiculars to C B from the points *f*, *h*, and *j* on the rim.

342. To make plain the reason of the power of the leverage being as here stated, we will employ Figs. 64 to 68 to represent in detail the five different positions we have assumed for the working cord, the same letters being used.

343. Figures 64 and 68 are simple enough; B C of Fig. 64 is as 2 to 1 for A B; and A B of Fig. 68 is as 1 to nothing for the leverage of the power p at B.

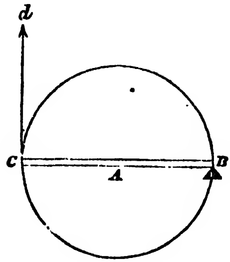


Fig. 64.

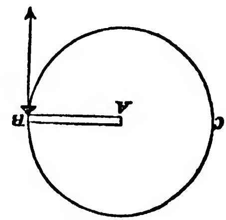


Fig. 68.

344. In Fig. 65 we assume the points B, A, and k, to be the three points of a triangular frame, with the power, and load, and fulcrum as shown.

This arrangement serves merely to show the balance between load and power; because here, where the fulcrum is fixed, the load A would rise at a rate nearly as much slower than the motion of the power in the direction $k\,e$, as the radius B A is less than the radius B k; whereas in the case of the toothed wheel upon a rack, the motion of the rim of the wheel must be equal to the motion of the cord; and as the rim rolls upon the rack, it follows that, as the wheel and cord-drum are of equal diameter, the wheel must rise bodily at the same rate of motion as the cord, and carry the centre A and the fulcrum B along with it; but the strain upon the cord varies, as we have shown, according to the angle of the pull.

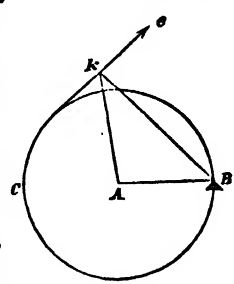


Fig. 65.

345. In Fig. 66 we have the power-leverage B l simply

equal to the load-leverage B A, so that the two forces balanced.

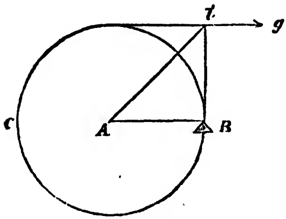


Fig. 66.

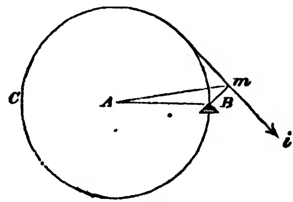


Fig. 67.

346. In Fig. 67 the power-leverage B *m* is only a fraction of the load-leverage B A, so that as B A is 1·0, the strain in the cord *m i* requires to be as much greater than the load A, as the leverage B A is greater than B *m*.

B *m* is shown sloping outward, as B *k* is shown sloping inward; but this inclination does not affect the result, because the power-lines are tangents to the pulley.

In Fig. 68 we have made B *n* represent the leverage of B *m* in Fig. 67; the ratio of leverage is the same in either case, that is, B *n* and B *m* are equal, and are merely representative lines, so that it matters not whether we show the power exerted on the outside or on the inside of the fulcrum B, so long as we keep its leverage in the same ratio with the load leverage.

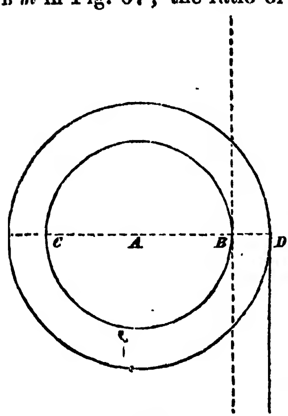


Fig. 69.

347. In Fig. 69 we apply the power on a diameter greater than that of the toothed wheel. In this case B D is the leverage of the power, and B A the leverage of the load; so that the power

must be as much greater than the load as B A is greater than B D.

Fig. 70 represents the action of the leverage in Fig. 69.

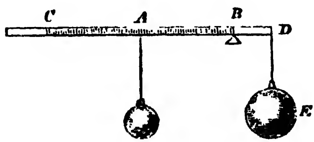


Fig. 70.

348. Fig. 71 shows a pulley resting on its axle *a*, with a load *c* borne on one side of the rim, to be raised by power in a cord which does not form a tangent to the circle at the other side, but is pulled in the oblique direction *b e* across the face of the pulley, and must, therefore, be supposed to be attached to a pin on the rim at *b*.

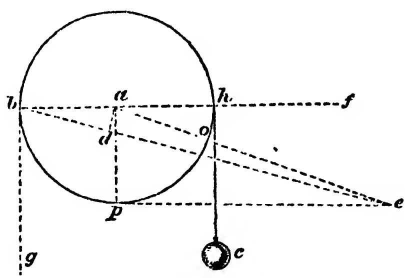


Fig. 71.

It is plain that were the cord to lead in the direction *b f* it would be powerless for motion, and would only bring pressure upon the axle *a*; and, as it would have full power equal to the force employed were it leading in the vertical direction *b g*, it follows that the moving power in the direction *b e* must be intermediate between no power in *b f* and full power in *b g*.

The line *b g* has the leverage equal to *a b*, acting against

the equal leverage $a\,h$ for the load; and as $a\,b$ is at right angles to the pull $b\,g$, so must $a\,d$ be at right angles to the pull $b\,e$, the fulcrum of the leverage being here at the centre of the axle a. We have, therefore, the leverage of the power in $b\,e$, equal to $a\,d$, acting against the leverage $a\,h$ of the load; so that, before the power can balance the load, it must be as much greater than the load as $a\,h$ is greater than $a\,d$.

The power here in the line $b\,e$ is acting with no more effect than the equal power of the weight i in Fig. 72,

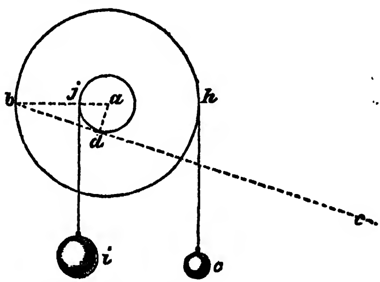


Fig. 72.

suspended by a cord on the smaller diameter, the radius $a\,j$ of which is just equal to the leverage $a\,d$ of Fig. 71.

849. If the cord lead away outward from the pin b in the direction $b\,l$, as in Fig. 73, we continue the line to m, and from k in the line $b\,m$ drop the perpendicular $k\,a$; then the power at l requires to be as much greater than the load c as $a\,h$ is greater than $a\,k$; and the effect of the power is the same as if the cord were upon a smaller diameter, with a radius equal to $a\,k$.

But supposing the points e in Fig. 71 and l in Fig. 73 fixed, the power in the line $l\,b$ will cease to have any effect upon the load c on the pin b descending to n, because then the point k is on the fulcrum a, and, therefore, has lost all leverage for motion. Whereas, the power in the

line *b e* of Fig. 71 does not cease its effect upon the load until the pin *b* has passed round to *o*, so as to bring *e*, *b*, and the fulcrum *a* into one straight line.

The power is decreasing in its passage from *b* to *n* in

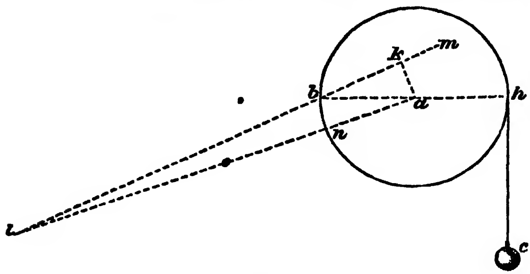


Fig. 73.

Fig. 73; whereas in Fig. 71 it increases till *b* reaches the point *p*, where it is at the maximum, the radius *a p* being there at right angles to the cord *p e*, which consequently forms a tangent to the circle at *p*. On passing *p*, the power decreases till, as we have said, it is lost at *o*.

350. Table of Natural Sines, Cosines, &c., see page 180.

Degrees.	Sine.	Co-versine.	Cosecant	Tangt.	Co-tangent.	Secant.	Ver-sine.	Cosine.	Degrees.
0	·00000	1·00000	Infinite.	·00000	Infinite.	1·00000	·0000	1 00000	90
1	01745	·98254	57 2986	·01745	57 2899	1·00015	.0001	99984	89
2	·03489	·96510	28 6537	·03492	28 6362	1·00061	·0006	·99939	88
3	05233	·94766	19 1073	·05240	19·0811	1·00137	·0013	·99863	87
4	·06975	·93024	14·3355	·06992	14 3006	1·00244	·0024	·99756	86
5	·08715	·91284	11·4737	·08748	11·4300	1·00382	·0038	·99619	85
6	·10452	·89547	9 56677	·10510	9 51436	1·00551	0054	99452	84
7	·12186	·87813	8 20550	.12278	8·14434	1·00751	0074	·99254	83
8	·13917	.86082	7 18529	·14054	7·11536	1·00983	·0097	·99026	82
9	15643	.84356	6 39245	·15838	6·31375	1 01247	·0123	98768	81
10	17364	·82635	5 75877	·17632	5·67128	1·01543	·0151	98480	80
11	19080	·80919	5·24084	·19438	5·14455	1·01872	·0183	·98162	79
12	20791	·79208	4·80973	·21255	4 70463	1·02234	·0218	97814	78
13	22495	·77504	4·44541	·23086	4 33147	1 02630	·0256	·97437	77
14	·24192	·75807	4·13356	·24932	4·01078	1 03061	·0297	·97029	76
15	25881	·74118	3·86370	26794	3 73205	1·03528	0340	96592	75
16	27563	·72436	3·62795	28674	3 48741	1·04030	·0387	96126	74
17	·29237	70762	3 42030	·30573	3 27085	1·04569	0436	·95630	73
18	·30901	69098	3 23606	32492	3 07768	1·05146	·0489	·95105	72
19	32556	67443	3·07155	·34432	2 90421	1·05762	·0544	94551	71
20	34202	·65797	2 92380	·36397	2 74747	1·06418	0603	93969	70
21	·35836	64163	2 79042	·38386	2 60508	1 07115	·0664	·93358	69
22	37460	·62539	2 66946	40402	2 47508	1 07853	·0728	92718	68
23	39073	·60926	2·55930	42447	2·35585	1 08636	0794	92050	67
24	·40673	·59326	2 45859	·44522	2 24603	1 09464	·0864	91354	66
25	42261	·57738	2 36620	·46630	2·14450	1·10338	0936	90630	65
26	·43837	·56162	2 28117	·48773	2 05030	1·11260	1012	89879	64
27	45399	·54600	2 20268	·50952	1 96261	1 12233	·1089	·89100	63
28	46947	·53052	2 13005	53170	1·88072	1 13257	1170	·88294	62
29	·48481	·51519	2 06266	·55430	1·80404	1·14335	1253	·87462	61
30	50000	50000	2 000 0	·57735	1 73205	1·15470	·1339	·86602	60
31	51503	·48496	1 94160	·60086	1 66427	1·16663	·1428	·85716	59
32	·52991	·47008	1 88707	62486	1 60033	1 17918	·1519	84804	58
33	54463	·45536	1 83607	·64940	1·53986	1·19236	·1613	83867	57
34	55919	·44080	1 78829	67450	1·48256	1 20622	·1709	·82903	56
35	·57357	·42642	1 74344	·70020	1 42814	1 22077	1808	81915	55
36	58778	·41221	1 70130	·72654	1 37638	1·23607	·1909	·80901	54
37	·60181	·39818	1 66164	75355	1 32704	1·25214	2013	79863	53
38	61566	·38433	1 62426	78128	1·27994	1·26902	·2119	·78801	52
39	62932	37067	1 58901	80978	1·23489	1·28676	·2228	·77714	51
40	·64278	·35721	1 55572	·83910	1 19175	1·30541	·2339	76604	50
41	65605	·34394	1 52425	·86928	1·15036	1 32501	·2452	·75471	49
42	·66913	·33086	1·49447	·90040	1·11061	1·34563	·2568	74314	48
43	·68199	31800	1 46627	·93251	1·07236	1·36733	·2686	·73135	47
44	·69465	·30534	1·43955	96568	1·03553	1·39016	·2806	71934	46
45	·70710	·29289	1·41421	1·00000	1·00000	1·41421	·2928	·70710	45
	Cosine	Versine.	Secant.	Co-tangent.	Tangent.	Cosecant.	Co-versine	Sine.	

INDEX.

SECTION I.

SECTION II.

SECTION III.

SECTION IV.

SECTION V.

SECTION VI.

SECTION X.

SECTION XI.

SECTION XII.

SECTION XIII.

SECTION XIV.

PRIZE MEDAL, INTERNATIONAL ——— was awarded to the Publishers of "Weale's Series."

7, *Stationers' Hall Court,*
May, 1871.

NEW LIST OF WEALE'S RUDIMENTARY, SCIENTIFIC, EDUCATIONAL, AND CLASSICAL SERIES,

OF WORKS SUITABLE FOR

Engineers, Architects, Builders, Artisans, and Students generally, as well as to those interested in Workmen's Libraries, Free Libraries, Literary and Scientific Institutions, Colleges, Schools, Science Classes, &c., &c.

*** THE ENTIRE SERIES IS FREELY ILLUSTRATED WHERE REQUISITE.

(*The Volumes contained in this List are bound in limp cloth, except where otherwise stated.*)

AGRICULTURE.

66. CLAY LANDS AND LOAMY SOILS, by J. Donaldson. 1s.
140. SOILS, MANURES, AND CROPS, by R. Scott Burn. 2s.
141. FARMING, AND FARMING ECONOMY, Historical and Practical, by R. Scott Burn. 3s.
142. CATTLE, SHEEP, AND HORSES, by R. Scott Burn. 2s. 6d.
145. MANAGEMENT OF THE DAIRY—PIGS—POULTRY, by R. Scott Burn. With Notes on the Diseases of Stock, by a Veterinary Surgeon. [illegible]
146. UTILISATION OF TOWN SEWAGE—IRRIGATION—RECLAMATION OF WASTE LAND, by R. Scott Burn. 2s. 6d.

Nos. 140, 141, 142, 145, and 146 bound in 2 vols., cloth boards, 14s.

LOCKWOOD & CO., 7, STATIONERS' HALL COURT.

ARCHITECTURE AND BUILDING.

16. **ARCHITECTURE**, Orders of, by W. H. Leeds. 1s.
17. ——————— Styles of, by T. Talbot Bury. 1s. 6d. } 1 vol., 2s. 6d.
18. ——————— Principles of Design, by E. L. Garbett. 2s.

Nos. 16, 17, and 18 in 1 vol. cloth boards, 5s. 6d.

22. **BUILDING**, the Art of, by E. Dobson. 1s. 6d.
23. **BRICK AND TILE MAKING**, by E. Dobson. 3s.
25. **MASONRY AND STONE-CUTTING**, by E. Dobson. New Edition, with Appendix on the Preservation of Stone. 2s. 6d.
30. **DRAINAGE AND SEWAGE OF TOWNS AND BUILDINGS**, by G. D. Dempsey. 2s.

With No. 29 (See page 4), Drainage of Districts and Lands, 3s.

35. **BLASTING AND QUARRYING OF STONE**, &c., by Field-Marshal Sir J. F. Burgoyne. 1s. 6d.
36. **DICTIONARY OF TECHNICAL TERMS** used by Architects, Builders, Engineers, Surveyors, &c. 4s.; cloth boards, 5s.
42. **COTTAGE BUILDING**, by C. B. Allen. 1s.
44. **FOUNDATIONS AND CONCRETE WORKS**, by E. Dobson. 1s. 6d.
45. **LIMES, CEMENTS, MORTARS, CONCRETE, MASTICS**, &c., by G. R. Burnell, C.E. 1s. 6d.
57. **WARMING AND VENTILATION**, by C. Tomlinson, F.R.S. 3s.
83**. **DOOR LOCKS AND IRON SAFES**, by Tomlinson. 2s. 6d.
111. **ARCHES, PIERS, AND BUTTRESSES**, by W. Bland. 1s. 6d.
116. **ACOUSTICS OF PUBLIC BUILDINGS**, by T. R. Smith. 1s. 6d.
123. **CARPENTRY AND JOINERY**, founded on Robison and Tredgold. 1s. 6d.
123*. **ILLUSTRATIVE PLATES** to the preceding. 4to. 4s. 6d.
124. **ROOFS FOR PUBLIC AND PRIVATE BUILDINGS**, founded on Robison, Price, and Tredgold. 1s. 6d.
127. **ARCHITECTURAL MODELLING IN PAPER**, Practical Instructions, by T. A. Richardson, Architect. 1s. 6d.
128. **VITRUVIUS'S ARCHITECTURE**, translated by J. Gwilt, with Plates. 5s.
130. **GRECIAN ARCHITECTURE**, Principles of Beauty in, by the Earl of Aberdeen. 1s.

Nos. 128 and 130 in 1 vol. cloth boards, 7s.

132. **ERECTION OF DWELLING-HOUSES**, with Specifications, Quantities of Materials, &c., by S. H. Brooks, 27 Plates. 2s. 6d.
156. **QUANTITIES AND MEASUREMENTS**. With Rules for Abstracting, &c. By A. C. Beaton. 1s. 6d.

ARITHMETIC AND MATHEMATICS.

MATHEMATICAL INSTRUMENTS, THEIR CONSTRUCTION, USE, &c., by J. F. Heather. Original Edition in 1 vol. 1*s*. 6*d*.

In ordering the above, be careful to say "Original Edition," to distinguish it from the Enlarged Edition in 3 vols., advertised on page 4 as in preparation.

60. LAND AND ENGINEERING SURVEYING, by T. Baker. 2*s*.
61*. READY RECKONER for the Admeasurement and Valuation of Land, by A. Arman. 1*s*. 6*d*.
76. GEOMETRY, DESCRIPTIVE, with a Theory of Shadows and Perspective, and a Description of the Principles and Practice of Isometrical Projection, by J. F. Heather. 2*s*.
83. COMMERCIAL BOOK-KEEPING, by James Haddon. 1*s*.
84. ARITHMETIC, with numerous Examples, by J. R. Young. 1*s*. 6*d*.
84*. KEY TO THE ABOVE, by J. R. Young. 1*s*. 6*d*.
85. EQUATIONAL ARITHMETIC: including Tables for the Calculation of Simple Interest, with Logarithms for Compound Interest, and Annuities, by W. Hipsley. 1*s*.
85*. SUPPLEMENT TO THE ABOVE, 1*s*.

85 and 85 in 1 vol., 2s.*

86. ALGEBRA, by J. Haddon. 2*s*.
86*. KEY AND COMPANION to the above, by J. R. Young. 1*s*. 6*d*.
88. THE ELEMENTS OF EUCLID, with Additional Propositions, and Essay on Logic, by H. Law. 2*s*.
90. ANALYTICAL GEOMETRY AND CONIC SECTIONS, by J. Hann. Entirely New Edition, improved and re-written by J. R. Young. [*In preparation.*
91. PLANE TRIGONOMETRY, by J. Hann. 1*s*.
92. SPHERICAL TRIGONOMETRY, by J. Hann. 1*s*.

Nos. 91 and 92 in 1 vol., 2s.

93. MENSURATION, by T. Baker. 1*s*. 6*d*.
94. MATHEMATICAL TABLES, LOGARITHMS, with Tables of Natural Sines, Cosines, and Tangents, by H. Law, C.E. 2*s*. 6*d*.
101. DIFFERENTIAL CALCULUS, by W. S. B. Woolhouse. 1*s*.
101*. WEIGHTS, MEASURES, AND MONEYS OF ALL NATIONS; with the Principles which determine the Rate of Exchange, by W. S. B. Woolhouse. 1*s*. 6*d*.
102. INTEGRAL CALCULUS, RUDIMENTS, by H. Cox, B.A. 1*s*.
103. INTEGRAL CALCULUS, Examples on, by J. Hann. 1*s*.
104. DIFFERENTIAL CALCULUS, Examples, by J. Haddon. 1*s*.
105. ALGEBRA, GEOMETRY, and TRIGONOMETRY, in Easy Mnemonical Lessons, by the Rev. T. P. Kirkman. 1*s*. 6*d*.
117. SUBTERRANEOUS SURVEYING, AND THE MAGNETIC VARIATION OF THE NEEDLE, by T. Fenwick, with Additions by T. Baker. 2*s*. 6*d*.

131 READY-RECKONER FOR MILLERS, FARMERS, AND MERCHANTS, showing the Value of any Quantity of Corn, with the Approximate Values of Mill-stones & Mi[illegible] 1s.

136. RUDIMENTARY ARITHMETIC, by J. Haddon, edited by A. Arman. 1s. 6d.

137. KEY TO THE ABOVE, by A. Arman. 1s. 6d.

147. STEPPING STONE TO ARITHMETIC, by A. Arman. 1s.

148. KEY TO THE ABOVE, by A. Arman. 1s.

158. THE SLIDE RULE, AND HOW TO USE IT. With Slide Rule in a pocket of cover. 3s.

*** *New Volumes in preparation :—*

DRAWING AND MEASURING INSTRUMENTS. Including—Instruments employed in Geometrical and Mechanical Drawing, the Construction, Copying, and Measurement of Maps, Plans, &c., by J. F. Heather, M.A. [*Just ready.*

OPTICAL INSTRUMENTS, more especially Telescopes, Microscopes, and Apparatus for producing copies of Maps and Plans by Photography, by J. F. Heather, M.A. [*Just ready.*

SURVEYING AND ASTRONOMICAL INSTRUMENTS. Including—Instruments Used for Determining the Geometrical Features of a portion of Ground, and in Astronomical Observations, by J. F. Heather, M.A. [*Just ready.*

*** *The above three volumes form an enlargement of the Author's original work, "Mathematical Instruments," the Tenth Edition of which (No. 32) is still on sale, price 1s. 6d.*

PRACTICAL PLANE GEOMETRY: Giving the Simplest Modes of Constructing Figures contained in one Plane, by J. F. Heather, M.A.

PROJECTION, Orthographic, Topographic, and Perspective: giving the various modes of Delineating Solid Forms by Constructions on a Single Plane Surface, by J. F. Heather, M.A.

*** *The above two volumes, with the Author's work already in the Series, "Descriptive Geometry," will form a complete Elementary Course of Mathematical Drawing.*

CIVIL ENGINEERING.

13. CIVIL ENGINEERING, by H. Law and G. R. Burnell. Fifth Edition, with Additions. 5s.

29. DRAINAGE OF DISTRICTS AND LANDS, by G. D. Dempsey. 1s. 6d.

With No. 30 (See page 2), Drainage and Sewage of Towns, 3s.

31. WELL-SINKING, BORING, AND PUMP WORK, by J. G. Swindell, revised by G. R. Burnell. 1*s*.
43. TUBULAR AND IRON GIRDER BRIDGES, including the Britannia and Conway Bridges, by G. D. Dempsey. 1*s*. 6*d*.
46. ROAD-MAKING AND MAINTENANCE OF MACADAMISED ROADS, by Field-Marshal Sir J. F. Burgoyne. 1*s*. 6*d*.
47. LIGHTHOUSES, their Construction and Illumination, by Alan Stevenson. 3*s*.
62. RAILWAY CONSTRUCTION, by Sir M. Stephenson. With Additions by E. Nugent, C.E. 2*s*. 6*d*.
62*. RAILWAY CAPITAL AND DIVIDENDS, with Statistics of Working, by E. D. Chattaway. 1*s*.
No. 62 and 62 in 1 vol., 3s. 6d.*
80*. EMBANKING LANDS FROM THE SEA, by J. Wiggins. 2*s*.
82**. GAS WORKS, and the PRACTICE of MANUFACTURING and DISTRIBUTING COAL GAS, by S. Hughes. 3*s*.
82***. WATER-WORKS FOR THE SUPPLY OF CITIES AND TOWNS, by S. Hughes, C.E. 3*s*.
118. CIVIL ENGINEERING OF NORTH AMERICA, by D. Stevenson. 3*s*.
120. HYDRAULIC ENGINEERING, by G. R. Burnell. 3*s*.
121. RIVERS AND TORRENTS, with the Method of Regulating their Course and Channels, Navigable Canals, &c., from the Italian of Paul Frisi. 2*s*. 6*d*.

EMIGRATION.

154. GENERAL HINTS TO EMIGRANTS. 2*s*.
157. EMIGRANT'S GUIDE TO NATAL, by R. J. Mann, M.D. 2*s*.
159. EMIGRANT'S GUIDE TO NEW SOUTH WALES, WESTERN AUSTRALIA, SOUTH AUSTRALIA, VICTORIA, AND QUEENSLAND, by James Baird, B.A. 2*s*. 6*d*.
160. EMIGRANT'S GUIDE TO TASMANIA AND NEW ZEALAND, by James Baird, B.A. [*In Preparation.*

FINE ARTS.

20. PERSPECTIVE, by George Pyne. 2*s*.
27. PAINTING; or, A GRAMMAR OF COLOURING, by G. Field. 2*s*.
40. GLASS STAINING, by Dr. M. A. Gessert, with an Appendix on the Art of Enamel Painting, &c. 1*s*.
41. PAINTING ON GLASS, from the German of Fromberg. 1*s*.
69. MUSIC, Treatise on, by C. C. Spencer. 2*s*.
71. THE ART OF PLAYING THE PIANOFORTE, by C. C. Spencer. 1*s*.

LEGAL TREATISES.

50. LAW OF CONTRACTS FOR WORKS AND SERVICES, by David Gibbons. 1s. 6d.
107. THE COUNTY COURT GUIDE, by a Barrister. 1s. 6d.
108. METROPOLIS LOCAL MANAGEMENT ACTS. 1s. 6d.
108*. METROPOLIS LOCAL MANAGEMENT AMENDMENT ACT, 1862; with Notes and Index. 1s.
Nos. 108 and 108 in 1 vol., 2s. 6d.*
109. NUISANCES REMOVAL AND DISEASES PREVENTION AMENDMENT ACT. 1s.
110. RECENT LEGISLATIVE ACTS applying to Contractors, Merchants, and Tradesmen. 1s.
151. THE LAW OF FRIENDLY, PROVIDENT, BUILDING, AND LOAN SOCIETIES, by N. White. 1s.
163. THE LAW OF PATENTS FOR INVENTIONS, by F. W. Campin, Barrister. 2s.

MECHANICS & MECHANICAL ENGINEERING.

6. MECHANICS, by Charles Tomlinson. 1s. 6d.
12. PNEUMATICS, by Charles Tomlinson. New Edition. 1s. 6d.
33. CRANES AND MACHINERY FOR RAISING HEAVY BODIES, the Art of Constructing, by J. Glynn. 1s.
34. STEAM ENGINE, by Dr. Lardner. 1s.
59. STEAM BOILERS, their Construction and Management, by R. Armstrong. With Additions by R. Mallet. 1s. 6d.
63. AGRICULTURAL ENGINEERING, BUILDINGS, MOTIVE POWERS, FIELD MACHINES, MACHINERY AND IMPLEMENTS, by G. H. Andrews, C.E. 3s.
67. CLOCKS, WATCHES, AND BELLS, by E. B. Denison. New Edition, with Appendix. 3s. 6d.
Appendix (to the 4th and 5th Editions) separately, 1s.
77*. ECONOMY OF FUEL, by T. S. Prideaux. 1s. 6d.
78*. THE LOCOMOTIVE ENGINE, by G. D. Dempsey. 1s. 6d.
79*. ILLUSTRATIONS TO THE ABOVE. 4to. 4s. 6d.
80. MARINE ENGINES, AND STEAM VESSELS, AND THE SCREW, by Robert Murray, C.E., Engineer Surveyor to the Board of Trade. Fifth Edition, revised and augmented, with a Glossary of Technical Terms, with their equivalents in French, German, and Spanish. 3s.
82. WATER POWER, as applied to Mills, &c., by J. Glynn. 2s.
97. STATICS AND DYNAMICS, by T. Baker. New Edition. 1s. 6d.
98. MECHANISM AND MACHINE TOOLS, by T. Baker; and TOOLS AND MACHINERY, by J. Nasmyth. 2s. 6d.
113*. MEMOIR ON SWORDS, by Col. Marey, translated by Lieut.-Col. H. H. Maxwell. 1s.

114. MACHINERY, Construction and Working, by C. D. Abel. 1s. 6d.
114*. PLATES TO THE ABOVE. 4to. 7s. 6d.
125. COMBUSTION OF COAL, AND THE PREVENTION OF SMOKE, by C. Wye Williams, M.I.C.E. 3s.
139. STEAM ENGINE, Mathematical Theory of, by T. Baker. 1s.
155. ENGINEER'S GUIDE TO THE ROYAL AND MERCANTILE NAVIES, by a Practical Engineer. Revised by D. F. McCarthy. 3s.
162. THE BRASSFOUNDER'S MANUAL, by W. Graham. 2s. 6d.
164. MODERN WORKSHOP PRACTICE. By J. G. Winton. 3s.

*** *New Volumes in preparation :—*

IRON AND HEAT, Exhibiting the Principles concerned in the Construction of Iron Beams, Pillars, and Bridge Girders, and the Action of Heat in the Smelting Furnace, by James Armour, C.E. Woodcuts. [*Just ready.*

POWER IN MOTION: Horse Power, Motion, Toothed Wheel Gearing, Long and Short Driving Bands, Angular Forces, &c., by James Armour, C.E. With 73 Diagrams. [*In the press.*

THE APPLICATION of IRON to BUILDING STRUCTURES, &c., by Francis Campin, C.E. With Illustrations. [*In the press.*

THE WORKMAN'S MANUAL OF ENGINEERING DRAWING, by John Maxton, Instructor in Engineering Drawing, Royal School of Naval Architecture and Marine Engineering, South Kensington. Plates and Diagrams. [*In the press.*

MINING TOOLS. For the Use of Mine Managers, Agents, Mining Students, &c., by William Morgans, Lecturer on Practical Mining at the Bristol School of Mines. 12mo. With an Atlas of Plates, containing 200 Illustrations. 4to. [*In the press.*

TREATISE ON THE METALLURGY OF IRON; containing Outlines of the History of Iron Manufacture, Methods of Assay, and Analysis of Iron Ores, Processes of Manufacture of Iron and Steel, &c., by H. Bauerman, F.G.S., A.R.S.M. Second Edition, revised and enlarged. Numerous Woodcuts.

NAVIGATION AND SHIP-BUILDING.

51. NAVAL ARCHITECTURE, by J. Peake. 3s.
53*. SHIPS FOR OCEAN AND RIVER SERVICE, Construction of, by Captain H. A. Sommerfeldt. 1s.
53**. ATLAS OF 15 PLATES TO THE ABOVE, Drawn for Practice. 4to. 7s. 6d.
54. MASTING, MAST-MAKING, and RIGGING OF SHIPS, by R. Kipping. 1s. 6d.

54*. IRON SHIP-BUILDING, by J. Grantham. Fifth Edition, with Supplement. 4s.

54**. ATLAS OF 40 PLATES to illustrate the preceding, including numerous Modern Examples—"The Warrior," "Hercules," "Bellerophon," &c. 4to. 38s.

55. NAVIGATION; the Sailor's Sea Book: How to Keep the Log and Work it off, Law of Storms, &c., by J. Greenwood. 2s.

83 *bis*. SHIPS AND BOATS, Form of, by W. Bland. 1s. 6d.

99. NAUTICAL ASTRONOMY AND NAVIGATION, by J. R. Young. 2s.

100*. NAVIGATION TABLES, for Use with the above. 1s. 6d.

106. SHIPS' ANCHORS for all SERVICES, by G. Cotsell. 1s. 6d.

149. SAILS AND SAIL-MAKING, by R. Kipping, N.A. 2s. 6d.

PHYSICAL AND CHEMICAL SCIENCE.

1. CHEMISTRY, by Prof. Fownes. With Appendix on Agricultural Chemistry. New Edition, with Index. 1s.

2. NATURAL PHILOSOPHY, by Charles Tomlinson. 1s.

3. GEOLOGY, by Major-Gen. Portlock. New Edition, with Index. 1s. 6d.

4. MINERALOGY, by A. Ramsay, Jun. 3s.

7. ELECTRICITY, by Sir W. S. Harris. 1s. 6d.

7*. GALVANISM, ANIMAL AND VOLTAIC ELECTRICITY, by Sir W. S. Harris. 1s. 6d.

8. MAGNETISM, by Sir W. S. Harris. 3s. 6d.

Nos. 7, 7, and 8 in 1 vol. cloth boards, 7s. 6d.*

11. HISTORY AND PROGRESS OF THE ELECTRIC TELEGRAPH, by Robert Sabine, C.E., F.S.A. 3s.

72. RECENT AND FOSSIL SHELLS (A Manual of the Mollusca), by S. P. Woodward. With Appendix by Ralph Tate, F.G.S. 6s. 6d.; in cloth boards, 7s. 6d. Appendix separately, 1s.

79**. PHOTOGRAPHY, the Stereoscope, &c., from the French of D. Van Monckhoven, by W. H. Thornthwaite. 1s. 6d.

96. ASTRONOMY, by the Rev. R. Main. New and Enlarged Edition, with an Appendix on "Spectrum Analysis." 1s. 6d.

133. METALLURGY OF COPPER, by Dr. R. H. Lamborn. 2s.

134. METALLURGY OF SILVER AND LEAD, by Dr. R. H. Lamborn. 2s.

135. ELECTRO-METALLURGY, by A. Watt. 2s.

138. HANDBOOK OF THE TELEGRAPH, by R. Bond. New and enlarged Edition. 1s. 6d.

143. EXPERIMENTAL ESSAYS—On the Motion of Camphor and Modern Theory of Dew, by C. Tomlinson. 1s.

161. QUESTIONS ON MAGNETISM, ELECTRICITY, AND PRACTICAL TELEGRAPHY, by W. McGregor. 1s. 6d.

MISCELLANEOUS TREATISES.

12. DOMESTIC MEDICINE, by Dr. Ralph Gooding. 2*s.*
112*. THE MANAGEMENT OF HEALTH, by James Baird. 1*s.*
113. USE OF FIELD ARTILLERY ON SERVICE, by Taubert, translated by Lieut.-Col. H. H. Maxwell. 1*s.* 6*d.*
150. LOGIC, PURE AND APPLIED, by S. H. Emmens. 1*s.* 6*d.*
152. PRACTICAL HINTS FOR INVESTING MONEY: with an Explanation of the Mode of Transacting Business on the Stock Exchange, by Francis Playford, Sworn Broker. 1*s.*
153. LOCKE ON THE CONDUCT OF THE HUMAN UNDERSTANDING, Selections from, by S. H. Emmens. 2*s.*

NEW SERIES OF EDUCATIONAL WORKS.

1. ENGLAND, History of, by W. D. Hamilton. 5*s.*; cloth boards, 6*s.* (Also in 5 parts, price 1*s.* each.)
5. GREECE, History of, by W. D. Hamilton and E. Levien, M.A. 2*s.* 6*d.*; cloth boards, 3*s.* 6*d.*
7. ROME, History of, by E. Levien, M.A. 2*s.* 6*d.*; cloth boards, 3*s.* 6*d.*
9. CHRONOLOGY OF HISTORY, LITERATURE, ART, and Progress, from the earliest period to the present time. 2*s.* 6*d.*; cloth boards, 3*s.* 6*d.*
11. ENGLISH GRAMMAR, by Hyde Clarke, D.C.L. 1*s.*
11*. HANDBOOK OF COMPARATIVE PHILOLOGY, by Hyde Clarke, D.C.L. 1*s.*
12. ENGLISH DICTIONARY, containing above 100,000 words, by Hyde Clarke, D.C.L. 3*s.* 6*d.*; cloth boards, 4*s.* 6*d.*
——————, with Grammar. Cloth bds. 5*s.* 6*d.*
14. GREEK GRAMMAR, by H. C. Hamilton. 1*s.*
15. ——— DICTIONARY, by H. R. Hamilton. Vol. 1. Greek—English. 2*s.*
17. ——————— Vol. 2. English—Greek, 2*s.*
——— Complete in 1 vol. 4*s.*; cloth boards, 5*s.*
——————, with Grammar. Cloth boards, 6*s.*
19. LATIN GRAMMAR, by T. Goodwin, M.A. 1*s.*
20. ——— DICTIONARY, by T. Goodwin, M.A. Vol. 1. Latin—English. 2*s.*
22. ——————— Vol. 2. English—Latin. 1*s.* 6*d.*
——— Complete in 1 vol. 3*s.* 6*d.*; cloth boards, 4*s.* 6*d.*
——————, with Grammar. Cloth bds. 5*s.* 6*d.*
24. FRENCH GRAMMAR, by G. L. Strauss. 1*s.*

25. FRENCH DICTIONARY, by A. Elwes. Vol. 1. French—English. 1*s*.
26. ———— Vol. 2. English—French. 1*s*. 6*d*.
———— Complete in 1 vol. 2*s*. 6*d*.; cloth boards, 3*s*. 6*d*.
————, with Grammar. Cloth bds. 4*s*. 6*d*.
27. ITALIAN GRAMMAR, by A. Elwes. 1*s*.
28. ———— TRIGLOT DICTIONARY, by A. Elwes. Vol. 1. Italian—English—French. 2*s*.
30. ———— Vol. 2. English—French—Italian. 2*s*.
32. ———— Vol. 3. French—Italian—English. 2*s*.
———— Complete in 1 vol. Cloth boards, 7*s*. 6*d*.
————, with Grammar. Cloth bds. 8*s*. 6*d*.
34. SPANISH GRAMMAR, by A. Elwes. 1*s*.
35. ———— ENGLISH AND ENGLISH-SPANISH DICTIONARY, by A. Elwes. 4*s*.; cloth boards, 5*s*.
————, with Grammar. Cloth boards, 6*s*.
39. GERMAN GRAMMAR, by G. L. Strauss. 1*s*.
40. ———— READER, from best Authors. 1*s*.
41. ———— TRIGLOT DICTIONARY, by N. E. S. A. Hamilton. Vol. 1. English—German—French. 1*s*.
42. ———— Vol. 2. German—French—English. 1*s*.
43. ———— Vol. 3. French—German—English. 1*s*.
———— Complete in 1 vol. 3*s*.; cloth boards, 4*s*.
————, with Grammar. Cloth boards, 5*s*.
44. HEBREW DICTIONARY, by Dr. Bresslau. Vol. 1. Hebrew—English. 6*s*.
————, with Grammar. 7*s*.
46. ———— Vol. 2. English—Hebrew. 3*s*.
————Complete, with Grammar, in 2 vols. Cloth boards, 12*s*.
46*. ———— GRAMMAR, by Dr. Bresslau. 1*s*.
47. FRENCH AND ENGLISH PHRASE BOOK. 1*s*.
48. COMPOSITION AND PUNCTUATION, by J. Brenan. 1*s*.
49. DERIVATIVE SPELLING BOOK, by J. Rowbotham. 1*s*. 6*d*.
50. DATES AND EVENTS. A Tabular View of English History, with Tabular Geography, by Edgar H. Rand. [*In Preparation*.

NEW READING BOOKS,

Adapted to the Requirements of the Revised Code, entitled

THE SCHOOL MANAGERS' SERIES.

Edited by the Rev. A. R. GRANT, Rector of Hitcham, and Honorary Canon of Ely; formerly H.M. Inspector of Schools.

	s.	d.		s.	d.
FIRST STANDARD	0	3	FOURTH STANDARD . . .	0	10
SECOND "	0	6	FIFTH " . . .	1	0
THIRD "	0	8	SIXTH " . . .	1	2

GREEK AND LATIN CLASSICS,

With Explanatory Notes in English.

LATIN SERIES.

1. A NEW LATIN DELECTUS, with Vocabularies and Notes, by H. Young . . . . 1*s.*
2. CÆSAR. De Bello Gallico; Notes by H. Young . . 2*s.*
3. CORNELIUS NEPOS; Notes by H. Young . . . 1*s.*
4. VIRGIL. The Georgics, Bucolics, and Doubtful Poems; Notes by W. Rushton, M.A., and H. Young . 1*s.* 6*d.*
5. VIRGIL. Æneid; Notes by H. Young . . . . 2*s.*
6. HORACE. Odes, Epodes, and Carmen Seculare, by H. Young 1*s.*
7. HORACE. Satires and Epistles, by W. B. Smith, M.A. 1*s.* 6*d.*
8. SALLUST. Catiline and Jugurthine War; Notes by W. M. Donne, B.A. . . . . 1*s.* 6*d.*
9. TERENCE. Andria and Heautontimorumenos; Notes by the Rev. J. Davies, M.A. . . . . 1*s.* 6*d.*
10. TERENCE. Adelphi, Hecyra, and Phormio; Notes by the Rev. J. Davies, M.A. . . . . 2*s.*
11. TERENCE. Eunuchus, by Rev. J. Davies, M.A. . 1*s.* 6*d.*
   Nos. 9, 10, and 11 in 1 vol. cloth boards, 6s.
12. CICERO. Oratio Pro Sexto Roscio Amerino. Edited, with Notes, &c., by J. Davies, M.A. (*Just ready.*) . . 1*s.*
14. CICERO. De Amicitia, de Senectute, and Brutus; Notes by the Rev. W. B. Smith, M.A. . . . . . . 2*s.*
16. LIVY. Books i., ii., by H. Young . . . . . 1*s.* 6*d.*
16*. LIVY. Books iii., iv., v., by H. Young . . . . 1*s.* 6*d.*
17. LIVY. Books xxi., xxii., by W. B. Smith, M.A. . 1*s.* 6*d.*
19. CATULLUS, TIBULLUS, OVID, and PROPERTIUS, Selections from, by W. Bodham Donne . . . . . 2*s.*
20. SUETONIUS and the later Latin Writers, Selections from, by W. Bodham Donne . . . . 2*s.*
21. THE SATIRES OF JUVENAL, by T. H. S. Escott, M.A., of Queen's College, Oxford . . . . . . 1*s.* 6*d.*

GREEK SERIES.

1. A NEW GREEK DELECTUS, by H. Young . 1s.
2. XENOPHON. Anabasis, i. ii. iii., by H. Young . . 1s.
3. XENOPHON. Anabasis, iv. v. vi. vii., by H. Young . 1s.
4. LUCIAN. Select Dialogues, by H. Young . . . 1s.
5. HOMER. Iliad, i. to vi., by T. H. L. Leary, D.C.L. 1s. 6d.
6. HOMER. Iliad, vii. to xii., by T. H. L. Leary, D.C.L. 1s. 6d.
7. HOMER. Iliad, xiii. to xviii., by T. H. L. Leary, D.C.L. 1s. 6d.
8. HOMER. Iliad, xix. to xxiv., by T. H. L. Leary, D.C.L. 1s. 6d.
9. HOMER. Odyssey, i. to vi., by T. H. L. Leary, D.C.L. 1s. 6d.
10. HOMER. Odyssey, vii. to xii., by T. H. L. Leary, D.C.L. 1s. 6d.
11. HOMER. Odyssey, xiii. to xviii., by T. H. L. Leary, D.C.L. 1s. 6d.
12. HOMER. Odyssey, xix. to xxiv.; and Hymns, by T. H. L. Leary, D.C.L. . . . . 2s.
13. PLATO. Apologia, Crito, and Phædo, by J. Davies, M.A. 2s.
14. HERODOTUS, Books i. ii., by T. H. L. Leary, D.C.L. 1s. 6d.
15. HERODOTUS, Books iii. iv., by T. H. L. Leary, D.C.L. 1s. 6d.
16. HERODOTUS, Books v. vi. vii., by T. H. L. Leary, D.C.L. 1s. 6d.
17. HERODOTUS, Books viii. ix., and Index, by T. H. L. Leary, D.C.L. . . . . 1s. 6d.
18. SOPHOCLES. Œdipus Tyrannus, by H. Young . . 1s.
20. SOPHOCLES. Antigone, by J. Milner, B.A. . . . 2s.
23. EURIPIDES. Hecuba and Medea, by W. B. Smith, M.A. 1s. 6d.
26. EURIPIDES. Alcestis, by J. Milner, B.A. . . . 1s.
30. ÆSCHYLUS. Prometheus Vinctus, by J. Davies, M.A. . 1s.
32. ÆSCHYLUS. Septem contra Thebas, by J. Davies, M.A. 1s.
40. ARISTOPHANES. Acharnenses, by C. S. D. Townshend, M.A. . . . . 1s. 6d.
41. THUCYDIDES. Peloponnesian War. Book i., by H. Young 1s.
42. XENOPHON. Panegyric on Agesilaus, by Ll. F. W. Jewitt 1s. 6d.

LONDON, *April*, 1871.

A Catalogue

OF

NEW & STANDARD WORKS

IN

ENGINEERING, ARCHITECTURE, AGRICULTURE, MATHEMATICS, MECHANICS, SCIENCE, &c. &c.

PUBLISHED BY

LOCKWOOD & CO.,

7, STATIONERS'-HALL COURT, LUDGATE HILL, E.C.

ENGINEERING, SURVEYING, &c.

Humber's New Work on Water-Supply.

A COMPREHENSIVE TREATISE on the WATER-SUPPLY of CITIES and TOWNS. By WILLIAM HUMBER, Assoc. Inst. C.E., and M. Inst. M.E. Author of "Cast and Wrought Iron Bridge Construction," &c. &c. This work, it is expected, will contain about 50 Double Plates, and upwards of 300 pages of Text. Imp. 4to, half bound in morocco. [*In the press.*

*** *In accumulating information for this volume, the Author has been very liberally assisted by several professional friends, who have made this department of engineering their special study. He has thus been in a position to prepare a work which, within the limits of a single volume, will supply the reader with the most complete and reliable information upon all subjects, theoretical and practical, connected with water supply. Through the kindness of Messrs. Anderson, Bateman, Hawksley, Homersham, Baldwin Latham, Lawson, Milne, Quick, Rawlinson, Simpson, and others, several works, constructed and in course of construction, from the designs of these gentlemen, will be fully illustrated and described.*

AMONGST OTHER IMPORTANT SUBJECTS THE FOLLOWING WILL BE TREATED IN THE TEXT:—

Historical Sketch of the means that have been proposed and adopted for the Supply of Water.—Water and the Foreign Matter usually associated with it.—Rainfall and Evaporation.—Springs and Subterranean Lakes.—Hydraulics.—The Selection of Sites for Water Works.—Wells.—Reservoirs.—Filtration and Filter Beds.—Reservoir and Filter Bed Appendages.—Pumps and Appendages.—Pumping Machinery.—Culverts and Conduits, Aqueducts, Syphons, &c.—Distribution of Water.—Water Meters and general House Fittings.—Cost of Works for the Supply of Water.—Constant and Intermittent Supply.—Suggestions for preparing Plans, &c. &c., together with a Description of the numerous Works illustrated, viz:—Aberdeen, Bideford, Cockermouth, Dublin, Glasgow, Loch Katrine, Liverpool, Manchester, Rotherham, Sunderland, and several others; with copies of the Contract, Drawings, and Specification in each case.

Humber's Modern Engineering. First Series.

A RECORD of the PROGRESS of MODERN ENGINEERING, 1863. Comprising Civil, Mechanical, Marine, Hydraulic, Railway, Bridge, and other Engineering Works, &c. By WILLIAM HUMBER, Assoc. Inst. C.E., &c. Imp. 4to, with 36 Double Plates, drawn to a large scale, and Photographic Portrait of John Hawkshaw, C.E., F.R.S., &c. Price 3*l.* 3*s.* half morocco.

List of the Plates.

NAME AND DESCRIPTION.	PLATES.	NAME OF ENGINEER.
Victoria Station and Roof—L. B.& S. C. Rail.	1 to 8	Mr. R. Jacomb Hood, C.E.
Southport Pier	9 and 10	Mr. James Brunlees, C.E.
Victoria Station and Roof—L. C. & D. & G.W. Railways	11 to 15A	Mr. John Fowler, C.E.
Roof of Cremorne Music Hall	16	Mr. William Humber, C.E.
Bridge over G. N. Railway	17	Mr. Joseph Cubitt, C.E.
Roof of Station—Dutch Rhenish Railway	18 and 19	Mr. Euschedi, C.E.
Bridge over the Thames—West London Extension Railway	20 to 24	Mr. William Baker, C.E.
Armour Plates	25	Mr. James Chalmers, C.E.
Suspension Bridge, Thames	26 to 29	Mr. Peter W. Barlow, C.E.
The Allen Engine	30	Mr. G. T. Porter, M.E.
Suspension Bridge, Avon	31 to 33	Mr. John Hawkshaw, C.E. and W. H. Barlow, C.E.
Underground Railway	34 to 36	Mr. John Fowler, C.E.

With copious Descriptive Letterpress, Specifications, &c.

"Handsomely lithographed and printed. It will find favour with many who desire to preserve in a permanent form copies of the plans and specifications prepared for the guidance of the contractors for many important engineering works."—*Engineer.*

Humber's Modern Engineering. Second Series.

A RECORD of the PROGRESS of MODERN ENGINEERING, 1864; with Photographic Portrait of Robert Stephenson, C.E., M.P., F.R.S., &c. Price 3*l.* 3*s.* half morocco.

List of the Plates.

NAME AND DESCRIPTION.	PLATES.	NAME OF ENGINEER.
Birkenhead Docks, Low Water Basin	1 to 15	Mr. G. F. Lyster, C.E.
Charing Cross Station Roof—C. C. Railway.	16 to 18	Mr. Hawkshaw, C.E.
Digswell Viaduct—Great Northern Railway.	19	Mr. J. Cubitt, C.E.
Robbery Wood Viaduct—Great N. Railway.	20	Mr. J. Cubitt, C.E.
Iron Permanent Way	20a	—
Clydach Viaduct—Merthyr, Tredegar, and Abergavenny Railway	21	Mr. Gardner, C.E.
Ebbw Viaduct ditto ditto ditto	22	Mr. Gardner, C.E.
College Wood Viaduct—Cornwall Railway	23	Mr. Brunel.
Dublin Winter Palace Roof	24 to 26	Messrs. Ordish & Le Feuvre.
Bridge over the Thames—L. C. & D. Railw.	27 to 32	Mr. J. Cubitt, C.E.
Albert Harbour, Greenock	33 to 36	Messrs. Bell & Miller.

With copious Descriptive Letterpress, Specifications, &c.

"A *resumé* of all the more interesting and important works lately completed in Great Britain; and containing, as it does, carefully executed drawings, with full working details, will be found a valuable accessory to the profession at large."—*Engineer.*

"Mr. Humber has done the profession good and true service, by the fine collection of examples he has here brought before the profession and the public."—*Practical Mechanic's Journal.*

Humber's Modern Engineering. Third Series.

A RECORD of the PROGRESS of MODERN ENGINEERING, 1865. Imp. 4to, with 40 Double Plates, drawn to a large scale, and Photographic Portrait of J. R. M'Clean, Esq., late President of the Institution of Civil Engineers. Price 3*l.* 3*s.* half morocco.

List of Plates and Diagrams.

MAIN DRAINAGE, METROPOLIS.

NORTH SIDE.

Map showing Interception of Sewers.
Middle Level Sewer. Sewer under Regent's Canal.
Middle Level Sewer. Junction with Fleet Ditch.
Outfall Sewer. Bridge over River Lea. Elevation.
Outfall Sewer. Bridge over River Lea. Details.
Outfall Sewer. Bridge over River Lea. Details.
Outfall Sewer. Bridges over Marsh Lane, North Woolwich Railway, and Bow and Barking Railway Junction.
Outfall Sewer. Bridge over Bow and Barking Railway. Elevation.
Outfall Sewer. Bridge over Bow and Barking Railway. Details.
Outfall Sewer. Bridge over Bow and Barking Railway. Details.
Outfall Sewer. Bridge over East London Waterworks' Feeder. Elevation.
Outfall Sewer. Bridge over East London Waterworks' Feeder. Details.
Outfall Sewer. Reservoir. Plan.
Outfall Sewer. Reservoir. Section.
Outfall Sewer. Tumbling Bay and Outlet.
Outfall Sewer. Penstocks.

SOUTH SIDE.

Outfall Sewer. Bermondsey Branch.
Outfall Sewer. Bermondsey Branch.
Outfall Sewer. Reservoir and Outlet. Plan.

MAIN DRAINAGE, METROPOLIS, *continued*—

Outfall Sewer. Reservoir and Outlet. Details.
Outfall Sewer. Reservoir and Outlet. Details.
Outfall Sewer. Reservoir and Outlet. Details.
Outfall Sewer. Filth Hoist.
Sections of Sewers (North and South Sides).

THAMES EMBANKMENT.

Section of River Wall.
Steam-boat Pier, Westminster. Elevation.
Steam-boat Pier, Westminster. Details.
Landing Stairs between Charing Cross and Waterloo Bridges.
York Gate. Front Elevation.
York Gate. Side Elevation and Details.
Overflow and Outlet at Savoy Street Sewer. Details.
Overflow and Outlet at Savoy Street Sewer. Penstock.
Overflow and Outlet at Savoy Street Sewer. Penstock.
Steam-boat Pier, Waterloo Bridge. Elevation.
Steam-boat Pier, Waterloo Bridge. Details.
Steam-boat Pier, Waterloo Bridge. Details.
Junction of Sewers. Plans and Sections.
Gullies. Plans and Sections.
Rolling Stock.
Granite and Iron Forts.

With copious Descriptive Letterpress, Specifications, &c.

Opinions of the Press.

"Mr. Humber's works—especially his annual 'Record,' with which so many of our readers are now familiar—fill a void occupied by no other branch of literature. . . . . The drawings have a constantly increasing value, and whoever desires to possess clear representations of the two great works carried out by our Metropolitan Board will obtain Mr. Humber's last volume."—*Engineering.*

"No engineer, architect, or contractor should fail to preserve these records of works which, for magnitude, have not their parallel in the present day, no student in the profession but should carefully study the details of these great works, which he may be one day called upon to imitate."—*Mechanics' Magazine.*

"A work highly creditable to the industry of its author. . . . . The volume is quite an encyclopædia for the study of the student who desires to master the subject of municipal drainage on its scale of greatest development."—*Practical Mechanic's Journal.*

Humber's Modern Engineering. Fourth Series.

A RECORD of the PROGRESS of MODERN ENGINEERING, 1866. Imp. 4to, with 36 Double Plates, drawn to a large scale, and Photographic Portrait of John Fowler, Esq., President of the Institution of Civil Engineers. Price 3*l.* 3*s.* half-morocco.

List of the Plates and Diagrams.

NAME AND DESCRIPTION.	PLATES.	NAME OF ENGINEER.
Abbey Mills Pumping Station, Main Drainage, Metropolis	1 to 4	Mr. Bazalgette, C.E.
Barrow Docks	5 to 9	Messrs. M'Clean & Stillman, C.E.
Manquis Viaduct, Santiago and Valparaiso Railway	10, 11	Mr. W. Loyd, C.E.
Adams' Locomotive, St. Helen's Canal Railw.	12, 13	Mr. H. Cross, C.E.
Cannon Street Station Roof, Charing Cross Railway	14 to 16	Mr. J. Hawkshaw, C.E.
Road Bridge over the River Moka	17, 18	Mr. H. Wakefield, C.E.
Telegraphic Apparatus for Mesopotamia	19	Mr. Siemens, C.E.
Viaduct over the River Wye, Midland Railw.	20 to 22	Mr. W. H. Barlow, C.E.
St. Germans Viaduct, Cornwall Railway	23, 24	Mr. Brunel, C.E.
Wrought-Iron Cylinder for Diving Bell	25	Mr. J. Coode, C.E.
Millwall Docks	26 to 31	Messrs. J. Fowler, C.E., and William Wilson, C.E.
Milroy's Patent Excavator	32	Mr. Milroy, C.E.
Metropolitan District Railway	33 to 38	Mr. J. Fowler, Engineer-in-Chief, and Mr. T. M. Johnson, C.E.
Harbours, Ports, and Breakwaters	A to C	

The Letterpress comprises—

A concluding article on Harbours, Ports, and Breakwaters, with Illustrations and detailed descriptions of the Breakwater at Cherbourg, and other important modern works; an article on the Telegraph Lines of Mesopotamia; a full description of the Wrought-iron Diving Cylinder for Ceylon, the circumstances under which it was used, and the means of working it; full description of the Millwall Docks; &c., &c., &c.

Opinions of the Press.

"Mr. Humber's 'Record of Modern Engineering' is a work of peculiar value, as well to those who design as to those who study the art of engineering construction. It embodies a vast amount of practical information in the form of full descriptions and working drawings of all the most recent and noteworthy engineering works. The plates are excellently lithographed, and the present volume of the 'Record' is not a whit behind its predecessors."—*Mechanics' Magazine.*

"We gladly welcome another year's issue of this valuable publication from the able pen of Mr. Humber. The accuracy and general excellence of this work are well known, while its usefulness in giving the measurements and details of some of the latest examples of engineering, as carried out by the most eminent men in the profession, cannot be too highly prized."—*Artizan.*

"The volume forms a valuable companion to those which have preceded it, and cannot fail to prove a most important addition to every engineering library."—*Mining Journal.*

"No one of Mr. Humber's volumes was bad; all were worth their cost, from the mass of plates from well-executed drawings which they contained. In this respect, perhaps, this last volume is the most valuable that the author has produced."—*Practical Mechanics' Journal.*

Humber's Great Work on Bridge Construction.

A COMPLETE and PRACTICAL TREATISE on CAST and WROUGHT-IRON BRIDGE CONSTRUCTION, including Iron Foundations. In Three Parts—Theoretical, Practical, and Descriptive. By WILLIAM HUMBER, Assoc. Inst. C. E., and M. Inst. M. E. Third Edition, revised and much improved, with 115 Double Plates (20 of which now first appear in this edition), and numerous additions to the Text. In 2 vols. imp. 4to., price 6*l.* 16*s.* 6*d.* half-bound in morocco. [*Recently published.*

"A very valuable contribution to the standard literature of civil engineering. In addition to elevations, plans, and sections, large scale details are given, which very much enhance the instructive worth of these illustrations. No engineer would willingly be without so valuable a fund of information."—*Civil Engineer and Architect's Journal.*

"The First or Theoretical Part contains mathematical investigations of the principles involved in the various forms now adopted in bridge construction. These investigations are exceedingly complete, having evidently been very carefully considered and worked out to the utmost extent that can be desired by the practical man. The tables are of a very useful character, containing the results of the most recent experiments, and amongst them are some valuable tables of the weight and cost of cast and wrought-iron structures actually erected. The volume of text is amply illustrated by numerous woodcuts, plates, and diagrams: and the plates in the second volume do great credit to both draughtsmen and engravers. In conclusion, we have great pleasure in cordially recommending this work to our readers."—*Artizan.*

"Mr. Humber's stately volumes lately issued—in which the most important bridges erected during the last five years, under the direction of the late Mr. Brunel, Sir W. Cubitt, Mr. Hawkshaw, Mr. Page, Mr. Fowler, Mr. Hemans, and others among our most eminent engineers, are drawn and specified in great detail."—*Engineer.*

Weale's Engineer's Pocket-Book.

THE ENGINEER'S, ARCHITECT'S, and CONTRACTOR'S POCKET-BOOK (LOCKWOOD & CO.'S; formerly WEALE'S). Published Annually. In roan tuck, gilt edges, with 10 Copper-Plates and numerous Woodcuts. Price 6*s.*

"There is no work published by or without authority, for the use of the scientific branches of the services, which contains anything like the amount of admirably arranged, reliable, and useful information. It is really a most solid, substantial, and excellent work; and not a page can be opened by a man of ordinary intelligence which will not satisfy him that this praise is amply deserved."—*Army and Navy Gazette.*

"A vast amount of really valuable matter condensed into the small dimensions of a book which is, in reality, what it professes to be—a pocket-book. . . . . We cordially recommend the book to the notice of the managers of coal and other mines; to them it will prove a handy book of reference on a variety of subjects more or less intimately connected with their profession. It might also be placed with advantage in the hands of the subordinate officers in collieries."—*Colliery Guardian.*

"The assignment of the late Mr. Weale's '*Engineer's Pocket-Book*' to Messrs. Lockwood & Co. has by no means lowered the standard value of the work. It is too well known among those for whom it is specially intended, to need more from us than the observation that this continuation of Mr. Weale's series of Pocket Books well sustains the reputation the work has so long enjoyed. Every branch of engineering is treated of, and facts, figures, and data of every kind abound."—*Mechanics' Mag.*

"It contains a large amount of information peculiarly valuable to those for whose use it is compiled. We cordially commend it to the engineering and architectural professions generally."—*Mining Journal.*

"A multitude of useful tables, without reference to which the engineer, architec or contractor could scarcely get through a single day's work."—*Scientific Review.*

Barlow on the Strength of Materials, enlarged.

A TREATISE ON THE STRENGTH OF MATERIALS, with Rules for application in Architecture, the Construction of Suspension Bridges, Railways, &c.; and an Appendix on the Power of Locomotive Engines, and the effect of Inclined Planes and gradients. By PETER BARLOW, F.R.S., Mem. Inst. of France; of the Imp. and Royal Academies of St. Petersburgh and Brussels; of the Amer. Soc. Arts; and Hon. Mem. Inst. Civil Engineers. A New and considerably Enlarged Edition, revised by his Sons, P. W. BARLOW, F.R.S., Mem. Inst. C.E., and W. H. BARLOW, F.R.S., Mem. of Council Inst. C.E., to which are added a Summary of Experiments by EATON HODGKINSON, F.R.S., WILLIAM FAIRBAIRN, F.R.S., and DAVID KIRKALDY; an Essay (with Illustrations) on the effect produced by passing Weights over Elastic Bars, by the Rev. ROBERT WILLIS, M.A., F.R.S. And Formulæ for Calculating Girders, &c. The whole arranged and edited by WILLIAM HUMBER, Assoc. Inst. C.E., and Mem. Inst. M.E., Author of "A Complete and Practical Treatise on Cast and Wrought-Iron Bridge Construction," &c. &c. Demy 8vo, 400 pp., with 19 large Plates, and numerous woodcuts, price 18*s*. cloth.

Opinions of the Press.

"This edition has undergone considerable improvement, and has been brought down to the present date. It is one of the first books of reference in existence."—*Artizan.*

"Although issued as the sixth edition, the volume under consideration is worthy of being regarded, for all practical purposes, as an entirely new work . . . the book is undoubtedly worthy of the highest commendation, and of an honourable place in the library of every engineer."—*Mining Journal.*

"An increased value has been given to this very valuable work by the addition of a large amount of information, which cannot prove otherwise than highly useful to those who require to consult it. . . . . The arrangement and editing of this mass of information has been undertaken by Mr. Humber, who has most ably fulfilled a task requiring special care and ability to render it a success, which this edition most certainly is. He has given the finishing touch to the volume by introducing into it an interesting memoir of Professor Barlow, which tribute of respect, we are sure, will be appreciated by the members of the engineering profession."—*Mechanics' Magazine.*

"A book which no engineer of any kind can afford to be without. In its present form its former value is much increased."—*Colliery Guardian.*

"The best book on the subject which has yet appeared. . . . . We know of no work that so completely fulfils its mission. . . . . As a scientific work of the first class, it deserves a foremost place on the bookshelves of every civil engineer and practical mechanic."—*English Mechanic.*

"There is not a pupil in an engineering school, an apprentice in an engineer's or architect's office, or a competent clerk of works, who will not recognise in the scientific volume newly given to circulation, an old and valued friend. . . So far as the strength of timber is concerned, there is no greater authority than Barlow."—*Building News.*

"It is scarcely necessary for us to make any comment upon the first portion of the new volume. . . . . Valuable alike to the student, tyro, and experienced practitioner, it will always rank in future, as it has hitherto done, as the standard treatise upon this particular subject."—*Engineer.*

"The present edition offers some important advantages over previous ones. The additions are both extensive and valuable, comprising experiments by Hodgkinson on the strength of cast-iron; extracts from papers on the transverse strength of beams by W. H. Barlow; an article on the strength of columns; experiments by Fairbairn, on iron and steel plates, on the behaviour of girders subjected to the vibration of a changing load, and on various cast and wrought-iron beams; experiments by Kirkaldy, on wrought-iron and steel bars, and a short appendix of formulæ for ready application in computing the strains on bridges."—*Engineering.*

Strains, Formulæ & Diagrams for Calculation of.

A HANDY BOOK for the CALCULATION of STRAINS in GIRDERS and SIMILAR STRUCTURES, and their STRENGTH; consisting of Formulæ and Corresponding Diagrams, with numerous Details for Practical Application, &c. By WILLIAM HUMBER, Assoc. Inst. C.E., &c. Fcap. 8vo, with nearly 100 Woodcuts and 3 Plates, price 7*s.* 6*d.* cloth.

It is hoped that a small work, in a handy form, devoted entirely to Bridge and Girder Calculations, without giving more than is absolutely necessary for the complete solution of practical problems, will meet with ready acceptance from the engineering profession. One of the chief features of the present work is the extensive application of simply constructed DIAGRAMS *to the calculation of the strains on bridges and girders.*

"To supply a universally recognised want of simple formulæ, applicable to the varied problems to be met with in ordinary practice, Mr. Humber, whose works on modern engineering afford sufficient evidence of his qualifications for the task, has compiled his 'Handy Book.' The arrangement of the matter in this little volume is as convenient as it well could be. . . . . The system of employing diagrams as a substitute for complex computations is one justly coming into great favour, and in that respect Mr. Humber's volume is fully up to the times."—*Engineering.*

"The formulæ are neatly expressed, and the diagrams good."—*Athenæum.*

"That a necessity existed for the book is evident, we think; that Mr. Humber has achieved his design is equally evident. . . . . We heartily commend the really *handy* book to our engineer and architect readers."—*English Mechanic.*

"It is, in fact, what its name indicates, a *handy book*, ... giving no more than is absolutely necessary for the complete solution of practical problems."—*Colliery Guardian.*

"This capital little work will supply a want, often found by engineers, viz., of having the requisite formulæ for calculating strains in a complete form, and yet sufficiently portable to be carried in the pocket. . . . Almost every formula that could possibly be required, together with diagrams of strains, is put concisely, yet clearly, in a work of considerably less size than an engineering pocket book."—*Artisan.*

Strains.

THE STRAINS ON STRUCTURES OF IRONWORK; with Practical Remarks on Iron Construction. By F. W. SHEILDS, M. Inst. C.E. Second Edition, with 5 plates. Royal 8vo, 5*s.* cloth.

CONTENTS.—Introductory Remarks; Beams Loaded at Centre; Beams Loaded at unequal distances between supports; Beams uniformly Loaded; Girders with triangular bracing Loaded at centre; Ditto, Loaded at unequal distances between supports; Ditto, uniformly Loaded; Calculation of the Strains on Girders with triangular Basings; Cantilevers; Continuous Girders; Lattice Girders; Girders with Vertical Struts and Diagonal Ties; Calculation of the Strains on Ditto; Bow and String Girders; Girders of a form not belonging to any regular figure; Plate Girders; Apportionments of Material to Strain; Comparison of different Girders; Proportion of Length to Depth of Girders; Character of the Work; Iron Roofs.

Trigonometrical Surveying.

AN OUTLINE OF THE METHOD OF CONDUCTING A TRIGONOMETRICAL SURVEY, for the Formation of Geographical and Topographical Maps and Plans, Military Reconnaissance, Levelling, &c., with the most useful Problems in Geodesy and Practical Astronomy, and Formulæ and Tables for Facilitating their Calculation. By MAJOR-GENERAL FROME, R.E., Inspector-General of Fortifications, &c. Third Edition, revised and improved. With 10 Plates and 113 Woodcuts. Royal 8vo, 12*s.* cloth.

Hydraulics.

HYDRAULIC TABLES, CO-EFFICIENTS, and FORMULÆ for finding the Discharge of Water from Orifices, Notches, Weirs, Pipes, and Rivers. By JOHN NEVILLE, Civil Engineer, M.R.I.A. Second Edition, with extensive Additions, New Formulæ, Tables, and General Information on Rain-fall, Catchment-Basins, Drainage, Sewerage, Water Supply for Towns and Mill Power. With numerous Woodcuts, 8vo, 16s. cloth.

*** This work contains a vast number of different hydraulic formulæ, and the most extensive and accurate tables yet published for finding the mean velocity of discharge from triangular, quadrilateral, and circular orifices, pipes, and rivers; with experimental results and co-efficients; effects of friction; of the velocity of approach; and of curves, bends, contractions, and expansions; the best form of channel; the drainage effects of long and short weirs, and weir-basins; extent of back-water from weirs; contracted channels; catchment-basins; hydrostatic and hydraulic pressure; water-power, &c. &c.

Levelling.

A TREATISE on the PRINCIPLES and PRACTICE of LEVELLING; showing its Application to Purposes of Railway and Civil Engineering, in the Construction of Roads; with Mr. TELFORD'S Rules for the same. By FREDERICK W. SIMMS, F.G.S., M. Inst. C.E. Fifth Edition, very carefully revised, with the addition of Mr. LAW'S Practical Examples for Setting out Railway Curves, and Mr. TRAUTWINE'S Field Practice of Laying out Circular Curves. With 7 Plates and numerous Woodcuts. 8vo, 8s. 6d. cloth. *** TRAUTWINE on Curves, separate, price 5s.

"One of the most important text-books for the general surveyor, and there is scarcely a question connected with levelling for which a solution would be sought but that would be satisfactorily answered by consulting the volume."—*Mining Journal.*

"The text-book on levelling in most of our engineering schools and colleges."—*Engineer.*

"The publishers have rendered a substantial service to the profession, especially to the younger members, by bringing out the present edition of Mr. Simms's useful work."—*Engineering.*

Tunnelling.

PRACTICAL TUNNELLING; explaining in Detail the Setting out of the Works; Shaft Sinking and Heading Driving; Ranging the Lines and Levelling Under-Ground; Sub-Excavating, Timbering, and the construction of the Brickwork of Tunnels; with the Amount of Labour required for, and the Cost of the various Portions of the Work. By FREDK. W. SIMMS, F.R.A.S., F.G.S., M. Inst. C.E., Author of "A Treatise on the Principles and Practice of Levelling," &c. &c. Second Edition, revised by W. DAVIS HASKOLL, Civil Engineer, Author of "The Engineer's Field-Book," &c. &c. With 16 large folding Plates and numerous Woodcuts. Imperial 8vo, 1l. 1s. cloth.

Strength of Cast Iron, &c.

A PRACTICAL ESSAY on the STRENGTH of CAST IRON and OTHER METALS; intended for the Assistance of Engineers, Iron-Masters, Millwrights, Architects, Founders, Smiths, and others engaged in the Construction of Machines, Buildings, &c.; containing Practical Rules, Tables, and Examples, founded on a series of New Experiments; with an Extensive Table of the Properties of Materials. By the late THOMAS TREDGOLD, Mem. Inst. C.E., Author of "Elementary Principles of Carpentry," "History of the Steam-Engine," &c. Fifth Edition, much improved. Edited by EATON HODGKINSON, F.R.S.; to which are added EXPERIMENTAL RESEARCHES on the STRENGTH and OTHER PROPERTIES of CAST IRON; with the Development of New Principles, Calculations Deduced from them, and Inquiries Applicable to Rigid and Tenacious Bodies generally. By the EDITOR. The whole Illustrated with 9 Engravings and numerous Woodcuts. 8vo, 12*s.* cloth.

*** HODGKINSON'S EXPERIMENTAL RESEARCHES ON THE STRENGTH AND OTHER PROPERTIES OF CAST IRON may be had separately. With Engravings and Woodcuts. 8vo, price 6*s.* cloth.

The High-Pressure Steam Engine.

THE HIGH-PRESSURE STEAM ENGINE; an Exposition of its Comparative Merits, and an Essay towards an Improved System of Construction, adapted especially to secure Safety and Economy. By Dr. ERNST ALBAN, Practical Machine Maker, Plau, Mecklenberg. Translated from the German, with Notes, by Dr. POLE, F.R.S., M. Inst. C.E., &c. &c. With 28 fine Plates, 8vo, 16*s.* 6*d.* cloth.

"A work like this, which goes thoroughly into the examination of the high-pressure engine, the boiler, and its appendages, &c., is exceedingly useful, and deserves a place in every scientific library."—*Steam Shipping Chronicle.*

Tables of Curves.

TABLES OF TANGENTIAL ANGLES and MULTIPLES for setting out Curves from 5 to 200 Radius. By ALEXANDER BEAZELEY, M. Inst. C.E. Printed on 48 Cards, and sold in a cloth box, waistcoat-pocket size, price 3*s.* 6*d.*

"Each table is printed on a small card, which, being placed on the theodolite, leaves the hands free to manipulate the instrument—no small advantage as regards the rapidity of work. They are clearly printed, and compactly fitted into a small case for the pocket—an arrangement that will recommend them to all practical men."—*Engineer.*

"Very handy: a man may know that all his day's work must fall on two of these cards, which he puts into his own card-case, and leaves the rest behind."—*Athenæum.*

Laying Out Curves.

THE FIELD PRACTICE of LAYING OUT CIRCULAR CURVES for RAILROADS. By JOHN C. TRAUTWINE, C.E., of the United States (extracted from SIMMS'S Work on Levelling). 8vo, 5*s.* sewed.

Estimate and Price Book.

THE CIVIL ENGINEER'S AND CONTRACTOR'S ESTIMATE AND PRICE BOOK for Home or Foreign Service: in reference to Roads, Railways, Tramways, Docks, Harbours, Forts, Fortifications, Bridges, Aqueducts, Tunnels, Sewers, Waterworks, Gasworks, Stations, Barracks, Warehouses, &c. &c. &c. With Specifications for Permanent Way, Telegraph Materials, Plant, Maintenance, and Working of a Railway; and a Priced List of Machinery, Plant, Tools, &c., required in the execution of Public Works. By W. DAVIS HASKOLL, C.E. Plates and numerous Woodcuts. Published annually. Demy 8vo, cloth, 6*s.*

"As furnishing a variety of data on every conceivable want to civil engineers and contractors, this book has ever stood perhaps unrivalled."—*Architect*, Jan. 21, 1871.

"The care with which the particulars are arranged reflects credit upon the author, each subject being divided into tables under their own special heads, so that no difficulty arises in finding the exact thing one wants. The value of the work to the student and the experienced contractor is inestimable."—*Mechanic's Mag.*, Feb. 3.

"Mr. Haskoll has bestowed very great care upon the preparation of his estimates and prices, and the work is one which appears to us to be in every way deserving of confidence."—*Builder's Weekly Reporter*, Jan. 27, 1871.

"Mr. Haskoll's book will prove of the utmost possible utility to the profession. The particulars are equally adapted to all branches of engineering. The manner in which the specifications are given leaves nothing to be desired, and to many young engineers they will prove invaluable. Even in the hands of those having some experience the book will often serve to call attention to matters which in the haste of estimating might otherwise be forgotten. It is altogether a work which few practising engineers will care to be without."—*Mining Journal*, Feb. 11, 1871.

Surveying (Land and Marine).

LAND AND MARINE SURVEYING, in Reference to the Preparation of Plans for Roads and Railways, Canals, Rivers, Towns' Water Supplies, Docks and Harbours; with Description and Use of Surveying Instruments. By W. DAVIS HASKOLL, C.E., Author of "The Engineer's Field Book," "Examples of Bridge and Viaduct Construction," &c. Demy 8vo, price 12*s.* 6*d.* cloth, with 14 folding Plates, and numerous Woodcuts.

"'Land and Marine Surveying' is a most useful and well arranged book for the aid of a student. . . . . We can strongly recommend it as a carefully-written and valuable text-book."—*Builder*, July 14, 1868.

"He only who is master of his subject can present it in such a way as to make it intelligible to the meanest capacity. It is in this that Mr. Haskoll excels. He has knowledge and experience, and can so give expression to it as to make any matter on which he writes, clear to the youngest pupil in a surveyor's office. . . . . The work will be found a useful one to men of experience, for there are few such who will not get some good ideas from it; but it is indispensable to the young practitioner."—*Colliery Guardian*, May 9, 1868.

"A volume which cannot fail to prove of the utmost practical utility. . . . . It is one which may be safely recommended to all students who aspire to become clean and expert surveyors; and from the exhaustive manner in which Mr. Haskoll has placed his long experience at the disposal of his readers, there will henceforth be no excuse for the complaint that young practitioners are at a disadvantage, through the neglect of their seniors to point out the importance of minute details, since they can readily supply the deficiency by the study of the volume now under consideration."—*Mining Journal*, May 5, 1868.

Fire Engineering.

FIRES, FIRE-ENGINES, AND FIRE BRIGADES. With a History of Manual and Steam Fire-Engines, their Construction, Use, and Management; Remarks on Fire-Proof Buildings, and the Preservation of Life from Fire; Statistics of the Fire Appliances in English Towns; Foreign Fire Systems; Hints for the formation of, and Rules for, Fire Brigades; and an Account of American Steam Fire-Engines. By CHARLES F. T. YOUNG, C.E., Author of "The Economy of Steam Power on Common Roads," &c. With numerous Illustrations, Diagrams, &c., handsomely printed, 544 pp., demy 8vo, price 1*l.* 4*s.* cloth.

"A large well-filled and useful book upon a subject which possesses a wide and increasing public interest. . . . . To such of our readers as are interested in the subject of fires and fire apparatus we can most heartily commend this book. . . . . It is really the only English work we now have upon the subject."—*Engineering.*

"Mr. Young has proved by his present work that he is a good engineer, and possessed of sufficient literary energy to produce a very readable and interesting volume."—*Engineer.*

"Fire, above all the elements, is to be dreaded in a great city, and Mr. Young deserves hearty thanks for the elaborate pains, benevolent spirit, scientific knowledge, and lucid exposition he has brought to bear upon the subject; and his substantial book should meet with substantial success, for it concerns every one who has even a skin which is not fireproof."—*Illustrated London News.*

"A volume which must be regarded as the text-book of its subject, and which in point of interest and intrinsic value is second to no contribution to a special department of history with which we are acquainted. 'Fires, Fire-Engines, and Fire Brigades' is the production of an earnest and diligent writer who comes to the task he has undertaken with a thorough love of it, and a firm determination to do it justice. . . . . The style of the work is admirable. . . . . It has the surpassing merit of being thoroughly reliable."—*Insurance Record.*

"That Mr. Young's treatise is an exhaustive one will be admitted when we state that there does not appear to be anything within the scope of his comprehensive title that has been left unnoticed. An immense amount of the most varied information relating to the subject has been collected from every conceivable source, and goes to form a history full of abiding interest. Great credit is unquestionably due to Mr. Young for having brought before the public the results of his exploration in this hitherto untrodden field. We strongly recommend the book to the notice of all who are in any way interested in fires, fire-engines, or fire-brigades."—*Mechanics' Magazine.*

Earthwork, Measurement and Calculation of.

A MANUAL on EARTHWORK. By ALEX. J. S. GRAHAM, C.E., Resident Engineer, Forest of Dean Central Railway. With numerous Diagrams. 18mo, 2*s.* 6*d.* cloth.

"We can cordially recommend the work to the notice of our readers."—*Building News.*

"As a really handy book for reference, we know of no work equal to it; and the railway engineers and others employed in the measurement and calculation of earthwork will find a great amount of practical information very admirably arranged, and available for general or rough estimates, as well as for the more exact calculations required in the engineers' contractor's offices."—*Artizan.*

"The object of this little book is an investigation of all the principles requisite for the measurement and calculation of earthworks, and a consideration of the data necessary for such operations. The author has evidently bestowed much care in effecting this object, and points out with much clearness the results of his own observations, derived from practical experience. The subjects treated of are accompanied by well-executed diagrams and instructive examples."—*Army and Navy Gazette.*

Field-Book for Engineers.

THE ENGINEER'S, MINING SURVEYOR'S, and CONTRACTOR'S FIELD-BOOK. By W. DAVIS HASKOLL, Civil Engineer. Second Edition, much enlarged, consisting of a Series of Tables, with Rules, Explanations of Systems, and Use of Theodolite for Traverse Surveying and Plotting the Work with minute accuracy by means of Straight Edge and Set Square only; Levelling with the Theodolite, Casting out and Reducing Levels to Datum, and Plotting Sections in the ordinary manner; Setting out Curves with the Theodolite by Tangential Angles and Multiples with Right and Left-hand Readings of the Instrument; Setting out Curves without Theodolite on the System of Tangential Angles by Sets of Tangents and Offsets; and Earthwork Tables to 80 feet deep calculated for every 6 inches in depth. With numerous wood-cuts, 12mo, price 12*s.* cloth.

"A very useful work for the practical engineer and surveyor. Every person engaged in engineering field operations will estimate the importance of such a work and the amount of valuable time which will be saved by reference to a set of reliable tables prepared with the accuracy and fulness of those given in this volume."—*Railway News.*

"The book is very handy, and the author might have added that the separate tables of sines and tangents to every minute will make it useful for many other purposes, the genuine traverse tables existing all the same."—*Athenæum.*

"The work forms a handsome pocket volume, and cannot fail, from its portability and utility, to be extensively patronised by the engineering profession."—*Mining Journal.*

"We know of no better field-book of reference or collection of tables than that Mr. Haskoll has given."—*Artizan.*

"A series of tables likely to be very useful to many civil engineers."—*Building News.*

"A very useful book of tables for expediting field-work operations. . . . The present edition has been much enlarged."—*Mechanics' Magazine.*

"We strongly recommend this second edition of Mr. Haskoll's 'Field Book' to all classes of surveyors."—*Colliery Guardian.*

Railway Engineering.

THE PRACTICAL RAILWAY ENGINEER. A concise Description of the Engineering and Mechanical Operations and Structures which are combined in the Formation of Railways for Public Traffic; embracing an Account of the Principal Works executed in the Construction of Railways; with Facts, Figures, and Data, intended to assist the Civil Engineer in designing and executing the important details required. By G. DRYSDALE DEMPSEY, C.E. Fourth Edition, revised and greatly extended. With 71 double quarto Plates, 72 Woodcuts, and Portrait of GEORGE STEPHENSON. One large vol. 4to, 2*l.* 12*s.* 6*d.* cloth.

Harbours.

THE DESIGN and CONSTRUCTION of HARBOURS. By THOMAS STEVENSON, F.R.S.E., M.I.C.E. Reprinted and enlarged from the Article "Harbours," in the Eighth Edition of "The Encyclopædia Britannica." With 10 Plates and numerous Cuts. 8vo, 10*s.* 6*d.* cloth.

Bridge Construction in Masonry, Timber, and Iron.

EXAMPLES OF BRIDGE AND VIADUCT CONSTRUCTION OF MASONRY, TIMBER, AND IRON; consisting of 46 Plates from the Contract Drawings or Admeasurement of select Works. By W. DAVIS HASKOLL, C.E. Second Edition, with the addition of 554 Estimates, and the Practice of Setting out Works, illustrated with 6 pages of Diagrams. Imp. 4to, price 2*l.* 12*s.* 6*d.* half-morocco.

"One of the very few works extant descending to the level of ordinary routine, and treating on the common every-day practice of the railway engineer. . . . A work of the present nature by a man of Mr. Haskoll's experience, must prove invaluable to hundreds. The tables of estimates appended to this edition will considerably enhance its value."—*Engineering.*

"We must express our cordial approbation of the work just issued by Mr. Haskoll. . . . . Besides examples of the best and most economical forms of bridge construction, the author has compiled a series of estimates which cannot fail to be of service to the practical man. . . . The examples of bridges are selected from those of the most notable construction on the different lines of the kingdom, and their details may consequently be safely followed."—*Railway News.*

"A very valuable volume, and may be added usefully to the library of every young engineer."—*Builder.*

"An excellent selection of examples, very carefully drawn to useful scales of proportion."—*Artizan.*

Mathematical and Drawing Instruments.

A TREATISE ON THE PRINCIPAL MATHEMATICAL AND DRAWING INSTRUMENTS employed by the Engineer, Architect, and Surveyor. By FREDERICK W. SIMMS, F.G.S., M. Inst. C.E., Author of "Practical Tunnelling," &c. &c. Third Edition, with a Description of the Theodolite, together with Instructions in Field Work, compiled for the use of Students on commencing practice. With numerous Cuts. 12mo, price 3*s.* 6*d.* cloth.

Oblique Arches.

A PRACTICAL TREATISE ON THE CONSTRUCTION of OBLIQUE ARCHES. By JOHN HART. Third Edition, with Plates. Imperial 8vo, price 8*s.* cloth.

*** *The small remaining stock of this work, which has been unobtainable for some time, has just been purchased by* LOCKWOOD & CO.

Oblique Bridges.

A PRACTICAL and THEORETICAL ESSAY on OBLIQUE BRIDGES, with 13 large folding Plates. By GEO. WATSON BUCK, M. Inst. C.E. Second Edition, corrected by W. H. BARLOW, M. Inst. C.E. Imperial 8vo, 12*s.* cloth.

"The standard text-book for all engineers regarding skew arches, is Mr. Buck's treatise, and it would be impossible to consult a better."—*Engineer.*

"A very complete treatise on the subject, re-edited by Mr. Barlow, who has added to it a method of making the requisite calculations without the use of trigonometrical formulæ."—*Builder.*

Weale's Series of Rudimentary Works.

These highly popular and cheap Series of Books, now comprising upwards of Two Hundred and Fifty distinct Works in almost every department of Science, Art, and Education, are recommended to the notice of Engineers, Architects, Builders, Artizans, and Students generally, as well as to those interested in Workmen's Libraries, Free Libraries, Literary and Scientific Institutions, Colleges, Schools, Science Classes, &c., &c. Lists of the several Series may be had on application to LOCKWOOD & CO.

The following is a Selection of the Works on Civil Engineering:—

STEAM ENGINE. By DR. LARDNER. 1*s.*

TUBULAR AND IRON GIRDER BRIDGES, including the Britannia and Conway Bridges. By G. D. DEMPSEY. 1*s.* 6*d.*

STEAM BOILERS, their Construction and Management. By R. ARMSTRONG. With Additions. 1*s.* 6*d.*

RAILWAY CONSTRUCTION. By SIR M. STEPHENSON. *New Edition,* 2*s.* 6*d.*

STEAM ENGINE, Mathematical Theory of. By T. BAKER. 1*s.*

ENGINEER'S GUIDE TO THE ROYAL AND MERCANTILE NAVIES. By a Practical Engineer. Revised by D. F. MCCARTHY. 3*s.*

LIGHTHOUSES, their Construction and Illumination. By ALAN STEVENSON. 3*s.*

CRANES AND MACHINERY FOR RAISING HEAVY BODIES, the Art of Constructing. By J. GLYNN. 1*s.*

CIVIL ENGINEERING. By H. LAW and G. R. BURNELL. *New Edition,* 5*s.*

DRAINING DISTRICTS AND LANDS. By G. D. DEMPSEY. 1*s.* 6*d.* } The 2 vols. in 1, 3*s.*

DRAINING AND SEWAGE OF TOWNS AND BUILDINGS. By G. D. DEMPSEY. 2*s.*

WELL-SINKING, BORING, AND PUMP WORK. By J. G. SWINDELL; Revised by G. R. BURNELL. 1*s.*

ROAD-MAKING AND MAINTENANCE OF MACADAMISED ROADS. By GEN. SIR J. BURGOYNE. 1*s.* 6*d.*

AGRICULTURAL ENGINEERING, BUILDINGS, MOTIVE POWERS, FIELD MACHINES, MACHINERY AND IMPLEMENTS. By G. H. ANDREWS, C.E. 3*s.*

ECONOMY OF FUEL. By T. S. PRIDEAUX. 1*s.* 6*d.*

EMBANKING LANDS FROM THE SEA. By J. WIGGINS. 2*s.*

WATER POWER, as applied to Mills, &c. By J. GLYNN. 2*s.*

GAS WORKS, AND THE PRACTICE OF MANUFACTURING AND DISTRIBUTING COAL GAS. By S. HUGHES, C.E. 3*s.*

WATERWORKS FOR THE SUPPLY OF CITIES AND TOWNS. By S. HUGHES, C.E. 3*s.*

SUBTERRANEOUS SURVEYING, AND THE MAGNETIC VARIATION OF THE NEEDLE. By T. FENWICK, with Additions by T. BAKER. 2*s.* 6*d.*

CIVIL ENGINEERING OF NORTH AMERICA. By D. STEVENSON. 3*s.*

HYDRAULIC ENGINEERING. By G. R. BURNELL. 3*s.*

RIVERS AND TORRENTS, with the Method of Regulating their Course and Channels, Navigable Canals, &c., from the Italian of PAUL FRISI. 2*s.* 6*d.*

COMBUSTION OF COAL AND THE PREVENTION OF SMOKE. By C. WYE WILLIAMS, M.I.C.E. 3*s.*

WATER POWER, as applied to Mills, &c. By J. GLYNN. 2*s.*

MARINE ENGINES AND STEAM VESSELS AND THE SCREW. By ROBERT MURRAY, C.E. *Fifth Edition.* 3*s.*

ENGINEER'S GUIDE TO THE ROYAL AND MERCANTILE NAVIES. By a PRACTICAL ENGINEER. Revised by D. F. MCCARTHY.

ARCHITECTURE.

Construction.

THE SCIENCE of BUILDING: an Elementary Treatise on the Principles of Construction. Especially adapted to the Requirements of Architectural Students. By E. WYNDHAM TARN, M.A., Architect. Illustrated with 47 Wood Engravings. Demy 8vo, price 8*s*. 6*d*. cloth. [*Recently published.*

"A very valuable book, which we strongly recommend to all students."—*Builder.*

"A modest and valuable book of reference for the student. . . . The formulæ will be found perfectly intelligible and available by the class for whom they are intended."—*Athenæum.*

"While Mr. Tarn's valuable little volume is quite sufficiently scientific to answer the purposes intended, it is written in a style that will deservedly make it popular. The diagrams are numerous and exceedingly well executed, and the student who has been warned not to read small print at night may safely study the present volume, as the type is bold and clear, and the lettering of the equations and formulæ so separated from the immediate text, that they can be at once identified and distinguished. The treatise does credit alike to the author and the publisher."—*Engineer*, Feb. 17, 1871.

"No architectural student should be without this hand-book of constructional knowledge."—*Architect.*

"The book is very far from being a mere compilation; it is an able digest of information which is only to be found scattered through various works, and contains more really original writing than many putting forth far stronger claims to originality. . . . . Mr. Tarn has done his work exceedingly well, and he has produced a book which ought to earn him the thanks of all architectural students. The book is clearly printed in bold type, the wood-cuts are all well executed, and the work is made of a very convenient size for reference."—*Engineering.*

Villa Architecture.

A HANDY BOOK of VILLA ARCHITECTURE; being a Series of Designs for Villa Residences in various Styles. With Detailed Specifications and Estimates. By C. WICKES, Architect, Author of "The Spires and Towers of the Mediæval Churches of England," &c. First Series, consisting of 30 Plates; Second Series, 31 Plates. Complete in 1 vol., 4to, price 2*l*. 10*s*. half morocco. Either Series separate, price 1*l*. 7*s*. each, half morocco.

"The whole of the designs bear evidence of their being the work of an artistic architect, and they will prove very valuable and suggestive to architects, students, and amateurs."—*Building News.*

The Architect's Guide.

THE ARCHITECT'S GUIDE; or, Office and Pocket Companion for Engineers, Architects, Land and Building Surveyors, Contractors, Builders, Clerks of Works, &c. By W. DAVIS HASKOLL, C.E., R. W. BILLINGS, Architect, F. ROGERS, and P. THOMPSON. With numerous Experiments by G. RENNIE, C.E., &c. Woodcuts, 12mo, cloth, price 3*s*. 6*d*.

Vitruvius' Architecture.

THE ARCHITECTURE OF MARCUS VITRUVIUS POLLIO. Translated by JOSEPH GWILT, F.S.A., F.R.A.S. Numerous Plates. 12mo, cloth limp, price 5*s*.

The Young Architect's Book.

HINTS TO YOUNG ARCHITECTS; comprising Advice to those who, while yet at school, are destined to the Profession; to such as, having passed their pupilage, are about to travel; and to those who, having completed their education, are about to practise. By GEORGE WIGHTWICK, Architect, Author of "The Palace of Architecture," &c. &c. Second Edition. With numerous Woodcuts. 8vo, 7*s.*, extra cloth.

Drawing for Builders and Students.

PRACTICAL RULES ON DRAWING for the OPERATIVE BUILDER and YOUNG STUDENT in ARCHITECTURE. By GEORGE PYNE, Author of a "Rudimentary Treatise on Perspective for Beginners." With 14 Plates, 4to, 7*s.* 6*d.*, boards.

CONTENTS.—I. Practical Rules on Drawing—Outlines. II. Ditto—the Grecian and Roman Orders. III. Practical Rules on Drawing—Perspective. IV. Practical Rules on Light and Shade. V. Practical Rules on Colour, &c. &c.

Cottages, Villas, and Country Houses.

DESIGNS and EXAMPLES of COTTAGES, VILLAS, and COUNTRY HOUSES; being the Studies of several eminent Architects and Builders; consisting of Plans, Elevations, and Perspective Views; with approximate Estimates of the Cost of each. In 4to, with 67 plates, price 1*l.* 1*s.*, cloth.

Weale's Builder's and Contractor's Price Book.

THE BUILDER'S AND CONTRACTOR'S PRICE BOOK (LOCKWOOD & CO.'s, formerly WEALE's). Published Annually. Containing Prices for Work in all branches of the Building Trade, with Items numbered for easy reference, and an Appendix of Tables, Notes, and Memoranda, arranged to afford detailed information, commonly required in preparing Estimates, &c. Originally Edited by the late GEO. R. BURNELL, C.E., &c. 12mo, 4*s.*, cloth.

"A multitudinous variety of useful information for builders and contractors. . . . . With its aid the prices for all work connected with the building trade may be estimated."—*Building News.*

"Carefully revised, admirably arranged, and clearly printed, it offers at a glance a ready method of preparing an estimate or specification upon a basis that is unquestionable. A reliable book of reference in the event of a dispute between employer and employed."—*Engineer.*

"Well done and reliable. It is the duty of a just critic to point out where any improvement can be made in any work, but Mr. Burnell has anticipated all objections in his clearly-printed book. We therefore recommend it to all branches of the profession."—*English Mechanic.*

"Mr. Burnell has omitted nothing from this work that could tend to render it valuable to the builder or contractor."—*Mechanic's Magazine.*

"It must find its place on the table of every civil engineer, builder, and contractor, as a standard work of reference."—*Artizan.*

"Well done and reliable. It is the duty of a just critic to point out where any improvement can be made in any work, but Mr. Burnell has anticipated all objections in his clearly printed book. We therefore recommend it to all branches of the profession."—*English Mechanic.*

Handbook of Specifications.

THE HANDBOOK OF SPECIFICATIONS; or, Practical Guide to the Architect, Engineer, Surveyor, and Builder, in drawing up Specifications and Contracts for Works and Constructions. Illustrated by Precedents of Buildings actually executed by eminent Architects and Engineers. Preceded by a Preliminary Essay, and Skeletons of Specifications and Contracts, &c., &c., and explained by numerous Lithograph Plates and Woodcuts. By Professor THOMAS L. DONALDSON, President of the Royal Institute of British Architects, Professor of Architecture and Construction, University College, London, M.I.B.A., Member of the various European Academies of the Fine Arts. With A REVIEW OF THE LAW OF CONTRACTS, and of the Responsibilities of Architects, Engineers, and Builders. By W. CUNNINGHAM GLEN, Barrister-at-Law, of the Middle Temple. 2 vols., 8vo, with upwards of 1100 pp. of text, and 33 Lithographic Plates, cloth, 2*l.* 2*s.*

"In these two volumes of 1,100 pages (together), forty-four specifications of executed works are given, including the specifications for parts of the new Houses of Parliament, by Sir Charles Barry, and for the new Royal Exchange, by Mr. Tite, M.P. The latter, in particular, is a very complete and remarkable document. It embodies, to a great extent, as Mr. Donaldson mentions, 'the bill of quantities, with the description of the works,' and occupies more than 100 printed pages.

"Amongst the other known buildings, the specifications of which are given, are the Wiltshire Lunatic Asylum (Wyatt and Brandon); Tothill Fields Prison (R. Abraham); the City Prison, Holloway (Bunning); the High School, Edinburgh (Hamilton); Clothworkers' Hall, London (Angel); Wellington College, Sandhurst (J. Shaw); Houses in Grosvenor Square, and elsewhere; St. George's Church, Doncaster (Scott); several works of smaller size by the Author, including Messrs. Shaw's Warehouse in Fetter Lane, a very successful elevation; the Newcastle-upon-Tyne Railway Station (J. Dobson); new Westminster Bridge (Page); the High Level Bridge, Newcastle (R. Stephenson); various works on the Great Northern Railway (Brydone); and one French specification for Houses in the Rue de Rivoli, Paris (MM. Armand, Hittorff, Pellechet, and Rohault de Fleury, architects). The last is a very elaborate composition, occupying seventy pages. The majority of the specifications have illustrations in the shape of elevations and plans.

"We are most glad to have the present work. It is valuable as a record, and more valuable still as a book of precedents.

"About 140 pages of the second volume are appropriated to an exposition of the law in relation to the legal liabilities of engineers, architects, contractors, and builders, by Mr. W. Cunningham Glen, Barrister-at-law; intended rather for those persons than for the legal practitioner. Suffice it, in conclusion, to say in words what our readers will have gathered for themselves from the particulars we have given, that Donaldson's Handbook of Specifications must be bought by all architects."—*Builder.*

Mechanical Engineering.

A PRACTICAL TREATISE ON MECHANICAL ENGINEERING: comprising Metallurgy, Moulding, Casting, Forging, Tools, Workshop Machinery, Mechanical Manipulation, Manufacture of the Steam Engine, &c. &c. With an Appendix on the Analysis of Iron and Iron Ore, and Glossary of Terms. By FRANCIS CAMPIN, C.E. Illustrated with 91 Woodcuts and 28 Plates of Slotting, Shaping, Drilling, Punching, Shearing, and Riveting Machines—Blast, Refining, and Reverberatory Furnaces—Steam Engines, Governors, Boilers, Locomotives, &c. Demy 8vo, cloth, price 12*s.*

Grantham's Iron Ship-Building, enlarged.

ON IRON SHIP-BUILDING; with Practical Examples and Details. Fifth Edition. Imp. 4to, boards, enlarged from 24 to 40 Plates (21 quite new), including the latest Examples. Together with separate Text, 12mo, cloth limp, also considerably enlarged, By JOHN GRANTHAM, M. Inst. C.E., &c. Price 2*l.* 2*s.* complete.

Description of Plates.

1. Hollow and Bar Keels, Stem and Stern Posts.
2. Side Frames, Floorings, and Bilge Pieces.
3. Floorings *continued*—Keelsons, Deck Beams, Gunwales, and Stringers.
4. Gunwales *continued*—Lower Decks, and Orlop Beams.
4*a*. Gunwales and Deck Beam Iron.
5. Angle-Iron, T Iron, Z Iron, Bulb Iron, as Rolled for Building.
6. Rivets, shown in section, natural size; Flush and Lapped Joints, with Single and Double Riveting.
7. Plating, three plans; Bulkheads and Modes of Securing them.
8. Iron Masts, with Longitudinal and Transverse Sections.
9. Sliding Keel, Water Ballast, Moulding the Frames in Iron Ship Building, Levelling Plates.
10. Longitudinal Section, and Half-breadth Deck Plan of Large Vessels on a reduced Scale.
11. Midship Sections of Three Vessels.
12. *Large Vessel*, showing Details—*Fore End* in Section, and End View, with Stern Post, Crutches, &c.
13. *Large Vessel*, showing Details—*After End* in Section, with End View, Stern Frame for Screw, and Rudder.
14. *Large Vessel*, showing Details—*Midship Section*, half breadth.
15. *Machines* for Punching and Shearing Plates and Angle-Iron, and for Bending Plates; Rivet Hearth.
15*a*. Beam-Bending Machine, Independent Shearing, Punching and Angle-Iron Machine.
15*b*. Double Lever Punching and Shearing Machine, arranged for cutting Angle and T Iron, with Dividing Table and Engine.
16. *Machines.*—Garforth's Riveting Machine, Drilling and Counter-Sinking Machine.
16*a*. Plate Planing Machine.
17. *Air Furnace* for Heating Plates and Angle-Iron: Various Tools used in Riveting and Plating.
18. *Gunwale*; Keel and Flooring; Plan for Sheathing with Copper.
18*a*. Grantham's Improved Plan of Sheathing Iron Ships with Copper.
19. Illustrations of the Magnetic Condition of various Iron Ships.
20. Gray's Floating Compass and Binnacle, with Adjusting Magnets, &c.
21. Corroded Iron Bolt in Frame of Wooden Ship; Jointing Plates.
22–4. *Great Eastern*—Longitudinal Sections and Half-breadth Plans—Midship Section, with Details—Section in Engine Room, and Paddle Boxes.
25-6. Paddle Steam Vessel of Steel.
27. *Scangrough*—Paddle Vessel of Steel.
28–9. Proposed Passenger Steamer.
30. *Persian*—Iron Screw Steamer.
31. Midship Section of H.M. Steam Frigate, *Warrior*.
32. Midship Section of H.M. Steam Frigate, *Hercules*.
33. Stem, Stern, and Rudder of H.M. Steam Frigate, *Bellerophon*.
34. Midship Section of H.M. Troop Ship, *Serapis*.
35. Iron Floating Dock.

"An enlarged edition of an elaborately illustrated work."—*Builder*, July 11, 1868.

"This edition of Mr. Grantham's work has been enlarged and improved, both with respect to the text and the engravings being brought down to the present period. . . . The practical operations required in producing a ship are described and illustrated with care and precision."—*Mechanics' Magazine*, July 17, 1868.

"A thoroughly practical work, and every question of the many in relation to iron shipping which admit of diversity of opinion, or have various and conflicting personal interests attached to them, is treated with sober and impartial wisdom and good sense. . . . . As good a volume for the instruction of the pupil or student of iron naval architecture as can be found in any language."—*Practical Mechanic's Journal*, August, 1868.

"A very elaborate work. . . . It forms a most valuable addition to the history of iron shipbuilding, while its having been prepared by one who has made the subject his study for many years, and whose qualifications have been repeatedly recognised, will recommend it as one of practical utility to all interested in shipbuilding."—*Army and Navy Gazette*, July 11, 1868.

CARPENTRY, TIMBER, &c.

Tredgold's Carpentry, new & enlarged Edition.

THE ELEMENTARY PRINCIPLES OF CARPENTRY: a Treatise on the Pressure and Equilibrium of Timber Framing, the Resistance of Timber, and the Construction of Floors, Arches, Bridges, Roofs, Uniting Iron and Stone with Timber, &c. To which is added an Essay on the Nature and Properties of Timber, &c., with Descriptions of the Kinds of Wood used in Building; also numerous Tables of the Scantlings of Timber for different purposes, the Specific Gravities of Materials, &c. By THOMAS TREDGOLD, C.E. Edited by PETER BARLOW, F.R.S. Fifth Edition, corrected and enlarged. With 64 Plates (11 of which now first appear in this edition), Portrait of the Author, and several Woodcuts. In 1 large vol., 4to, 2*l.* 2*s.* extra cloth. [*Recently published.*

"'Tredgold's Carpentry' ought to be in every architect's and every builder's library, and those who do not already possess it ought to avail themselves of the new issue."—*Builder*, April 9, 1870.

"A work whose monumental excellence must commend it wherever skilful carpentry is concerned. The Author's principles are rather confirmed than impaired by time, and, as now presented, combine the surest base with the most interesting display of progressive science. The additional plates are of great intrinsic value."—*Building News*, Feb. 25, 1870.

"'Tredgold's Carpentry' has ever held a high position, and the issue of the fifth edition, in a still more improved and enlarged form, will give satisfaction to a very large number of artisans who desire to raise themselves in their business, and who seek to do so by displaying a greater amount of knowledge and intelligence than their fellow-workmen. It is as complete a work as need be desired. To the superior workman the volume will prove invaluable; it contains treatises written in language which he will readily comprehend."—*Mining Journal*, Feb. 12, 1870.

Grandy's Timber Tables.

THE TIMBER IMPORTER'S, TIMBER MERCHANT'S, and BUILDER'S STANDARD GUIDE. By RICHARD E. GRANDY. Comprising:—An Analysis of Deal Standards, Home and Foreign, with comparative Values and Tabular Arrangements for Fixing Nett Landed Cost on Baltic and North American Deals, including all intermediate Expenses, Freight, Insurance, Duty, &c., &c.; together with Copious Information for the Retailer and Builder. 12mo, price 7*s.* 6*d.* cloth.

"Everything it pretends to be: built up gradually, it leads one from a forest to a trenail, and throws in, as a makeweight, a host of material concerning bricks, columns, cisterns, &c.—all that the class to whom it appeals requires."—*English Mechanic.*

"The only difficulty we have is as to what is NOT in its pages. What we have tested of the contents, taken at random, is invariably correct."—*Illustrated Builder's Journal.*

Tables for Packing-Case Makers.

PACKING-CASE TABLES; showing the number of Superficial Feet in Boxes or Packing-Cases, from six inches square and upwards. Compiled by WILLIAM RICHARDSON, Accountant. Oblong 4to, cloth, price 3*s.* 6*d.*

"Will save much labour and calculation to packing-case makers and those who use packing-cases."—*Grocer.* "Invaluable labour-saving tables."—*Ironmonger.*

Nicholson's Carpenter's Guide.

THE CARPENTER'S NEW GUIDE; or, BOOK of LINES for CARPENTERS: comprising all the Elementary Principles essential for acquiring a knowledge of Carpentry. Founded on the late PETER NICHOLSON's standard work. A new Edition, revised by ARTHUR ASHPITEL, F.S.A., together with Practical Rules on Drawing, by GEORGE PYNE. With 74 Plates, 4to, 1*l.* 1*s.* cloth.

Dowsing's Timber Merchant's Companion.

THE TIMBER MERCHANT'S AND BUILDER'S COMPANION; containing New and Copious Tables of the Reduced Weight and Measurement of Deals and Battens, of all sizes, from One to a Thousand Pieces, and the relative Price that each size bears per Lineal Foot to any given Price per Petersburgh Standard Hundred; the Price per Cube Foot of Square Timber to any given Price per Load of 50 Feet; the proportionate Value of Deals and Battens by the Standard, to Square Timber by the Load of 50 Feet; the readiest mode of ascertaining the Price of Scantling per Lineal Foot of any size, to any given Figure per Cube Foot. Also a variety of other valuable information. By WILLIAM DOWSING, Timber Merchant. Second Edition. Crown 8vo, 3*s.* cloth.

"Everything is as concise and clear as it can possibly be made. There can be no doubt that every timber merchant and builder ought to possess it, because such possession would, with use, unquestionably save a very great deal of time, and, moreover, ensure perfect accuracy in calculations. There is also another class besides these who ought to possess it; we mean all persons engaged in carrying wood, where it is requisite to ascertain its weight. Mr. Dowsing's tables provide an easy means of doing this. Indeed every person who has to do with wood ought to have it."—*Hull Advertiser.*

MECHANICS, &c.

Mechanic's Workshop Companion.

THE OPERATIVE MECHANIC'S WORKSHOP COMPANION; and THE SCIENTIFIC GENTLEMAN'S PRACTICAL ASSISTANT; comprising a great variety of the most useful Rules in Mechanical Science; with numerous Tables of Practical Data and Calculated Results. By W. TEMPLETON, Author of "The Engineer's, Millwright's, and Machinist's Practical Assistant." Tenth Edition, with Mechanical Tables for Operative Smiths, Millwrights, Engineers, &c.; together with several Useful and Practical Rules in Hydraulics and Hydrodynamics, a variety of Experimental Results, and an Extensive Table of Powers and Roots. 11 Plates. 12mo, 5*s.* bound. [*Just published.*

"As a text-book of reference, in which mechanical and commercial demands are judiciously met, TEMPLETON'S COMPANION stands unrivalled."—*Mechanics' Magazine.*

"Admirably adapted to the wants of a very large class. It has met with great success in the engineering workshop, as we can testify; and there are a great many men who, in a great measure, owe their rise in life to this little work."—*Building News.*

Engineer's Assistant.

THE ENGINEER'S, MILLWRIGHT'S, and MACHINIST'S PRACTICAL ASSISTANT; comprising a Collection of Useful Tables, Rules, and Data. Compiled and Arranged, with Original Matter, by W. TEMPLETON. 4th Edition. 18mo, 2*s*. 6*d*. cloth.

"So much varied information compressed into so small a space, and published at a price which places it within the reach of the humblest mechanic, cannot fail to command the sale which it deserves. With the utmost confidence we commend this book to the attention of our readers."—*Mechanics' Magazine.*

"Every mechanic should become the possessor of the volume, and a more suitable present to an apprentice to any of the mechanical trades could not possibly be made."—*Building News.*

Designing, Measuring, and Valuing.

THE STUDENT'S GUIDE to the PRACTICE of MEASURING, and VALUING ARTIFICERS' WORKS; containing Directions for taking Dimensions, Abstracting the same, and bringing the Quantities into Bill, with Tables of Constants, and copious Memoranda for the Valuation of Labour and Materials in the respective Trades of Bricklayer and Slater, Carpenter and Joiner, Painter and Glazier, Paperhanger, &c. With 43 Plates and Woodcuts. Originally edited by EDWARD DOBSON, Architect. New Edition, re-written, with Additions on Mensuration and Construction, and several useful Tables for facilitating Calculations and Measurements. By E. WYNDHAM TARN, M.A., Architect. 8vo, 10*s*. 6*d*. cloth. [*Just published.*

"This useful book should be in every architect's and builder's office. It contains a vast amount of information absolutely necessary to be known."—*The Irish Builder.*

"The book is well worthy the attention of the student in architecture and surveying, as by the careful study of it his progress in his profession will be much facilitated."—*Mining Journal*, Feb. 11, 1871.

"We have failed to discover anything connected with the building trade, from excavating foundations to bell-hanging, that is not fully treated upon in this valuable work."—*The Artizan*, March, 1871.

"Mr. Tarn has well performed the task imposed upon him, and has made many further and valuable additions, embodying a large amount of information relating to the technicalities and modes of construction employed in the several branches of the building trade. . . . . . From the extent of the information which the volume embodies, and the care taken to secure accuracy in every detail, it cannot fail to prove of the highest value to students, whether training in the offices of provincial surveyors, or in those of London practitioners."—*Colliery Guardian*, February 10th, 1871.

"Altogether the book is one which well fulfils the promise of its title-page, and we can thoroughly recommend it to the class for whose use it has been compiled. Mr. Tarn's additions and revisions have much increased the usefulness of the work, and have especially augmented its value to students. Finally, it is only just to the publishers to add that the book has been got up in excellent style, the typography being bold and clear, and the plates very well executed."—*Engineering*, March 24, 1871.

Superficial Measurement.

THE TRADESMAN'S GUIDE TO SUPERFICIAL MEASUREMENT. Tables calculated from 1 to 200 inches in length, by 1 to 108 inches in breadth. For the use of Architects, Surveyors, Engineers, Timber Merchants, Builders, &c. By JAMES HAWKINGS. Fcp. 3*s*. 6*d*. cloth.

MATHEMATICS, &c.

Gregory's Practical Mathematics.

MATHEMATICS for PRACTICAL MEN; being a Commonplace Book of Pure and Mixed Mathematics. Designed chiefly for the Use of Civil Engineers, Architects, and Surveyors. Part I. PURE MATHEMATICS—comprising Arithmetic, Algebra, Geometry, Mensuration, Trigonometry, Conic Sections, Properties of Curves. Part II. MIXED MATHEMATICS—comprising Mechanics in general, Statics, Dynamics, Hydrostatics, Hydrodynamics, Pneumatics, Mechanical Agents, Strength of Materials. With an Appendix of copious Logarithmic and other Tables. By OLINTHUS GREGORY, LL.D., F.R.A.S. Enlarged by HENRY LAW, C.E. 4th Edition, carefully revised and corrected by J. R. YOUNG, formerly Professor of Mathematics, Belfast College; Author of "A Course of Mathematics," &c. With 13 Plates. Medium 8vo, 1*l.* 1*s.* cloth.

"As a standard work on mathematics it has not been excelled."—*Artizan.*

"The engineer or architect will here find ready to his hand, rules for solving nearly every mathematical difficulty that may arise in his practice. As a moderat acquaintance with arithmetic, algebra, and elementary geometry is absolutely necessary to the proper understanding of the most useful portions of this book, the author very wisely has devoted the first three chapters to those subjects, so that the most ignorant may be enabled to master the whole of the book, without aid from any other. The rules are in all case explained by means of examples, in which every step of the process is clearly worked out."—*Builder.*

"One of the most serviceable books to the practical mechanics of the country. . . . The edition of 1847 was fortunately entrusted to the able hands of Mr. Law, who revised it thoroughly, re-wrote many chapters, and added several sections to those which had been rendered imperfect by advanced knowledge. On examining the various and many improvements which he introduced into the work, they seem almost like a new structure on an old plan, or rather like the restoration of an old ruin, not only to its former substance, but to an extent which meets the larger requirements of modern times. . . . . In the edition just brought out, the work has again been revised by Professor Young. He has modernised the notation throughout, introduced a few paragraphs here and there, and corrected the numerous typographical errors which have escaped the eyes of the former Editor. The book is now as complete as it is possible to make it. . . . . We have carried our notice of this book to a greater length than the space allowed us justified, but the experiments it contains are so interesting, and the method of describing them so clear, that we may be excused for overstepping our limit. It is an instructive book for the student, and a Textbook for him who having once mastered the subjects it treats of, needs occasionally to refresh his memory upon them."—*Building News.*

The Metric System.

A SERIES OF METRIC TABLES, in which the British Standard Measures and Weights are compared with those of the Metric System at present in use on the Continent. By C. H. DOWLING, C. E. 8vo, 10*s.* 6*d.* strongly bound.

"Mr. Dowling's Tables, which are well put together, come just in time as a ready reckoner for the conversion of one system into the other."—*Athenæum.*

"Their accuracy has been certified by Professor Airy, the Astronomer Royal."—*Builder.*

"Resolution 8.—That advantage will be derived from the recent publication of Metric Tables, by C. H. Dowling, C.E."—*Report of Section F, British Association, Bath.*

Inwood's Tables, greatly enlarged and improved.

TABLES FOR THE PURCHASING of ESTATES, Freehold, Copyhold, or Leasehold; Annuities, Advowsons, &c., and for the Renewing of Leases held under Cathedral Churches, Colleges, or other corporate bodies; for Terms of Years certain, and for Lives; also for Valuing Reversionary Estates, Deferred Annuities, Next Presentations, &c., together with Smart's Five Tables of Compound Interest, and an Extension of the same to lower and Intermediate Rates. By WILLIAM INWOOD, Architect. The 18th edition, with considerable additions, and new and valuable Tables of Logarithms for the more Difficult Computations of the Interest of Money, Discount, Annuities, &c., by M. FÉDOR THOMAN, of the Société Crédit Mobilier of Paris. 12mo, 8s. cloth.

*** *This edition (the 18th) differs in many important particulars from former ones. The changes consist,* first, *in a more convenient and systematic arrangement of the original Tables, and in the removal of certain numerical errors which a very careful revision of the whole has enabled the present editor to discover; and* secondly, *in the extension of practical utility conferred on the work by the introduction of Tables now inserted for the first time. This new and important matter is all so much actually added to* INWOOD'S TABLES; *nothing has been abstracted from the original collection: so that those who have been long in the habit of consulting* INWOOD *for any special professional purpose will, as heretofore, find the information sought still in its pages.*

"Those interested in the purchase and sale of estates, and in the adjustment of compensation cases, as well as in transactions in annuities, life insurances, &c., will find the present edition of eminent service."—*Engineering.*

"More than half a century has elapsed since the first edition was published, yet 'Inwood's Tables' still maintain a most enviable reputation; and when it is considered that the new issue, the Eighteenth edition, has been enriched by large additional contributions by Mr. Fédor Thoman, of the French Crédit Mobilier, whose carefully arranged tables of logarithms for the more difficult computations of the interest of money, discount, annuities, &c., cannot fail to be of the utmost utility, its value will be appreciated. The introduction contains an admirable epitome of the principles of decimals, and an explanation of all that is necessary to render the elaborate tables in the book of thorough utility to all consulting it. This new edition will certainly be referred to with quite as much confidence as its predecessors."—*Mining Journal.*

Compound Interest and Annuities.

THEORY of COMPOUND INTEREST and ANNUITIES; with Tables of Logarithms for the more Difficult Computations of Interest, Discount, Annuities, &c., in all their Applications and Uses for Mercantile and State Purposes. With an elaborate Introduction. By FÉDOR THOMAN, of the Société Crédit Mobilier, Paris. 12mo, cloth, 5s.

"A very powerful work, and the Author has a very remarkable command of his subject."—*Professor A. de Morgan.*

"No banker, merchant, tradesman, or man of business, ought to be without Mr. Thoman's truly 'handy-book.'"—*Review.*

"The author of this 'handy-book' deserves our thanks."—*Insurance Gazette.*

"We recommend it to the notice of actuaries and accountants."—*Athenæum.*

SCIENCE AND ART

The Military Sciences.

AIDE-MÉMOIRE to the MILITARY SCIENCES. Framed from Contributions of Officers and others connected with the different Services. Originally edited by a Committee of the Corps of Royal Engineers. Second Edition, most carefully revised by an Officer of the Corps, with many additions; containing nearly 350 Engravings and many hundred Woodcuts. 3 vols. royal 8vo, extra cloth boards, and lettered, price 4*l.* 10*s.*

"A compendious encyclopædia of military knowledge, to which we are greatly indebted."—*Edinburgh Review.*

"The most comprehensive work of reference to the military and collateral sciences. Among the list of contributors, some seventy-seven in number, will be found names of the highest distinction in the services. . . . The work claims and possesses the great merit that by far the larger portion of its subjects have been treated originally by the practical men who have been its contributors."—*Volunteer Service Gazette.*

Field Fortification.

A TREATISE on FIELD FORTIFICATION, the ATTACK of FORTRESSES, MILITARY, MINING, and RECONNOITRING. By Colonel I. S. MACAULAY, late Professor of Fortification in the Royal Military Academy, Woolwich. Sixth Edition, crown 8vo, cloth, with separate Atlas of 12 Plates, sewed, price 12*s.* complete.

Dye-Wares and Colours.

THE MANUAL of COLOURS and DYE-WARES: their Properties, Applications, Valuation, Impurities, and Sophistications. For the Use of Dyers, Printers, Dry Salters, Brokers, &c. By J. W. SLATER. Post 8vo, cloth, price 7*s.* 6*d.* [*Recently published.*

"Essentially a manual for practical men, and precisely such a book as practical men will appreciate."—*Scientific Review.*

"A complete encyclopædia of the *materia tinctoria*. The information given respecting each article is full and precise, and the methods of determining the value of articles such as these, so liable to sophistication, are given with clearness, and are practical as well as valuable."—*Chemist and Druggist.*

Electricity.

A MANUAL of ELECTRICITY; including Galvanism, Magnetism, Diamagnetism, Electro-Dynamics, Magno-Electricity, and the Electric Telegraph. By HENRY M. NOAD, Ph.D., F.C.S., Lecturer on Chemistry at St. George's Hospital. Fourth Edition, entirely rewritten. Illustrated by 500 Woodcuts. 8vo, 1*l.* 4*s.* cloth.

"This publication fully bears out its title of 'Manual.' It discusses in a satisfactory manner electricity, frictional and voltaic, thermo-electricity, and electro-physiology."—*Athenæum.*

"The commendations already bestowed in the pages of the *Lancet* on the former editions of this work are more than ever merited by the present. The accounts given of electricity and galvanism are not only complete in a scientific sense, but, which is a rarer thing, are popular and interesting."—*Lancet.*

Text-Book of Electricity.

THE STUDENT'S TEXT-BOOK OF ELECTRICITY: including Magnetism, Voltaic Electricity, Electro-Magnetism, Diamagnetism, Magneto-Electricity, Thermo-Electricity, and Electric Telegraphy. Being a Condensed Résumé of the Theory and Application of Electrical Science, including its latest Practical Developments, particularly as relating to Aerial and Submarine Telegraphy. By HENRY M. NOAD, Ph.D., Lecturer on Chemistry at St. George's Hospital. Post 8vo, 400 Illustrations, 12*s.* 6*d.* cloth.

**** In carrying out the design of this work, the author has availed himself both of the matter (in a condensed form) and of the illustrations of his* "Manual of Electricity," *but the present volume will be found to contain much additional and important information, (with many new illustrations,) which has become available since the publication of his larger work.*

"We can recommend Dr. Noad's book for clear style, great range of subject, a good index, and a plethora of woodcuts. Such collections as the present are indispensable." —*Athenæum.*

"A most elaborate compilation of the facts of electricity and magnetism, and of the theories which have been advanced concerning them."—*Popular Science Review.*

"Clear, compendious, compact, well illustrated, and well printed, this is an excellent manual."—*Lancet.*

"We can strongly recommend the work, as an admirable text book, to every student—beginner or advanced—of electricity."—*Engineering.*

"The most complete manual on the subject of electricity to be met with."—*Observer.*

"Nothing of value has been passed over, and nothing given but what will lead to a correct, and even an exact, knowledge of the present state of electrical science."—*Mechanics' Magazine.*

"We know of no book or other treatise containing so much information on experimental fact as this does [illegible] and no book of any size that contains so complete a range of facts."—*English Mechanic.*

Chemical Analysis.

THE COMMERCIAL HANDBOOK of CHEMICAL ANALYSIS; or Practical Instructions for the determination of the Intrinsic or Commercial Value of Substances used in Manufactures, in Trades, and in the Arts. By A. NORMANDY, Author of "Practical Introduction to Rose's Chemistry," and Editor of Rose's "Treatise of Chemical Analysis." Illustrated with Woodcuts. Second and cheaper Edition, post 8vo, 9*s.* cloth.

"We recommend this book to the careful perusal of every one; it may be truly affirmed to be of universal interest, and we strongly recommend it to our readers as a guide, alike indispensable to the housewife as to the pharmaceutical practitioner."—*Medical Times.*

"A volume of surpassing interest, in which the Author describes the character and properties of 400 different articles of commerce, the substances by which they are too frequently adulterated, and the means of their detection."—*Mining Journal.*

"The very best work on the subject the English press has yet produced."—*Mechanics' Magazine.*

Practical Philosophy.

A SYNOPSIS of PRACTICAL PHILOSOPHY. By the Rev. JOHN CARR, M.A., late Fellow of Trin. Coll., Cambridge. Second Edition. 18mo, 5*s.* cloth.

Science and Art.

THE YEAR-BOOK of FACTS in SCIENCE and ART; exhibiting the most important Improvements and Discoveries of the Past Year in Mechanics and the Useful Arts, Natural Philosophy, Electricity, Chemistry, Zoology and Botany, Geology and Mineralogy, Meteorology and Astronomy. By JOHN TIMBS, F.S.A., Author of "Curiosities of Science," "Things not Generally Known," &c. With Steel Portrait and Vignette. Fcap. 5*s*. cloth.

*** *This work, published annually, records the proceedings of the principal scientific societies, and is indispensable to all who wish to possess a faithful record of the latest novelties in science and the arts.*

The back Volumes, from 1861 to 1870, each containing a Steel Portrait, and an extra Volume for 1862, with Photograph, may still be had, price 5*s*. each.

"Persons who wish for a concise annual summary of important scientific events will find their desire in the 'Year Book of Facts.'"—*Athenæum.*

"The standard work of its class. Mr. Timbs's 'Year Book' is always full of suggestive and interesting matter, and is an excellent *résumé* of the year's progress in the sciences and the arts."—*Builder.*

"A correct exponent of scientific progress . . . . a record of abiding interest. If anyone wishes to know what progress science has made, or what has been done in any branch of art during the past year, he has only to turn to Mr. Timbs's pages, and is sure to obtain the required information."—*Mechanics' Magazine.*

"An invaluable compendium of scientific progress for which the public are indebted to the untiring energy of Mr. Timbs."—*Atlas.*

"There is not a more useful or more interesting compilation than the 'Year Book of Facts.' . . . The discrimination with which Mr. Timbs selects his facts, and the admirable manner in which he condenses into a comparatively short space all the salient features of the matters which he places on record, are deserving of great praise."—*Railway News.*

Science and Scripture.

SCIENCE ELUCIDATIVE OF SCRIPTURE, AND NOT ANTAGONISTIC TO IT; being a Series of Essays on—1. Alleged Discrepancies; 2. The Theory of the Geologists and Figure of the Earth; 3. The Mosaic Cosmogony; 4. Miracles in general—Views of Hume and Powell; 5. The Miracle of Joshua—Views of Dr. Colenso: The Supernaturally Impossible; 6. The Age of the Fixed Stars—their Distances and Masses. By Professor J. R. YOUNG, Author of "A Course of Elementary Mathematics," &c. &c. Fcap. 8vo, price 5*s*. cloth lettered.

"Professor Young's examination of the early verses of Genesis, in connection with modern scientific hypotheses, is excellent."—*English Churchman.*

"Distinguished by the true spirit of scientific inquiry, by great knowledge, by keen logical ability, and by a style peculiarly clear, easy, and energetic."—*Nonconformist.*

"No one can rise from its perusal without being impressed with a sense of the singular weakness of modern scepticism."—*Baptist Magazine.*

"The author has displayed considerable learning and critical acumen in combating the objections alluded to. . . . . The volume is one of considerable value, inasmuch as it contains much sound thought, and is calculated to assist the reader to discriminate truth from error, at least so far as a finite mind is able to separate them. The work, therefore, must be considered to be a valuable contribution to controversial theological literature."—*City Press.*

Geology and Genesis Harmonised.

THE TWIN RECORDS of CREATION; or, Geology and Genesis, their Perfect Harmony and Wonderful Concord. By GEORGE W. VICTOR LE VAUX. With numerous Illustrations. Fcap. 8vo, price 5*s.* cloth.

"We can recommend Mr. Le Vaux as an able and interesting guide to a popular appreciation of geological science."—*Spectator.*

"The author combines an unbounded admiration of science with an unbounded admiration of the Written Record. The two impulses are balanced to a nicety; and the consequence is, that difficulties, which to minds less evenly poised, would be serious, find immediate solutions of the happiest kinds."—*London Review.*

"Mr. Le Vaux very feasibly reconciles the two records."—*Builder.*

"A most instructive and readable book. We welcome this volume as aiding in a most important discussion, and commend it to those interested in the subject."—*Evangelical Magazine.*

"Vigorously written, reverent in spirit, stored with instructive geological facts, and designed to show that there is no discrepancy or inconsistency between the Word and the works of the Creator. The future of Nature, in connexion with the glorious destiny of man, is vividly conceived."—*Watchman.*

"A valuable contribution to the evidences of Revelation, and disposes very conclusively of the arguments of those who would set God's works against God's Word. No real difficulty is shirked, and no sophistry is left unexposed."—*The Rock.*

Wood-Carving.

INSTRUCTIONS in WOOD-CARVING, for Amateurs; with Hints on Design. By A LADY. In emblematic wrapper, handsomely printed, with Ten large Plates, price 2*s.* 6*d.*

"The handicraft of the wood-carver, so well as a book can impart it, may be learnt from 'A Lady's' publication."—*Athenæum.*

"A real *practical guide.* It is very complete."—*Literary Churchman.*

"The directions given are plain and easily understood, and it forms a very good introduction to the practical part of the carver's art."—*English Mechanic.*

"The writer of the little treatise before us deals very plainly and practically with her subject. . . . . The illustrations are admirably drawn, nearly all the patterns being remarkable for their graceful ornamentation. We can confidently recommend the book to amateur wood-carvers."—*Young Englishwoman.*

Popular Work on Painting.

PAINTING POPULARLY EXPLAINED; with Historical Sketches of the Progress of the Art. By THOMAS JOHN GULLICK, Painter, and JOHN TIMBS, F.S.A. Second Edition, revised and enlarged. With Frontispiece and Vignette. In small 8vo, 6*s.* cloth.

*** This Work has been adopted as a Prize-book in the Schools of Art at South Kensington.

"A work that may be advantageously consulted. Much may be learned, even by those who fancy they do not require to be taught, from the careful perusal of this unpretending but comprehensive treatise."—*Art Journal.*

"A valuable book, which supplies a want. It contains a large amount of original matter, agreeably conveyed, and will be found of value, as well by the young artist seeking information as by the general reader. We give a cordial welcome to the book, and augur for it an increasing reputation."—*Builder.*

"This volume is one that we can heartily recommend to all who are desirous of understanding what they admire in a good painting."—*Daily News.*

Scott Burn's Introduction to Farming.

THE LESSONS of MY FARM: a Book for Amateur Agriculturists, being an Introduction to Farm Practice, in the Culture of Crops, the Feeding of Cattle, Management of the Dairy, Poultry, and Pigs, and in the Keeping of Farm-work Records. By ROBERT SCOTT BURN, Editor of "The Year-Book of Agricultural Facts," &c. With numerous Illustrations. Fcp. 6*s.* cloth.

"A most complete introduction to the whole round of farming practice."—*John Bull.*

"There are many hints in it which even old farmers need not be ashamed to accept."—*Morning Herald.*

Tables for Land Valuers.

THE LAND VALUER'S BEST ASSISTANT: being Tables, on a very much improved Plan, for Calculating the Value of Estates. To which are added, Tables for reducing Scotch, Irish, and Provincial Customary Acres to Statute Measure; also, Tables of Square Measure, and of the various Dimensions of an Acre in Perches and Yards, by which the Contents of any Plot of Ground may be ascertained without the expense of a regular Survey; &c. By R. HUDSON, Civil Engineer. New Edition, with Additions and Corrections, price 4*s.* strongly bound.

"This new edition includes tables for ascertaining the value of leases for any term of years; and for showing how to lay out plots of ground of certain acres in forms, square, round, &c., with valuable rules for ascertaining the probable worth of standing timber to any amount; and is of incalculable value to the country gentleman and professional man."—*Farmer's Journal.*

The Laws of Mines and Mining Companies.

A PRACTICAL TREATISE on the LAW RELATING to MINES and MINING COMPANIES. By WHITTON ARUNDELL, Attorney-at-Law. Crown 8vo. 4*s.* cloth.

Auctioneer's Assistant.

THE APPRAISER, AUCTIONEER, HOUSE AGENT, and HOUSE BROKER'S POCKET ASSISTANT. By JOHN WHEELER, Valuer. A new edition, much improved and enlarged, and entirely re-written. [*In the Press.*

The Civil Service Book-keeping.

BOOK-KEEPING NO MYSTERY: its Principles popularly explained, and the Theory of Double Entry analysed; for the use of Young Men Commencing Business, Examination Candidates, and Students generally. By an EXPERIENCED BOOK-KEEPER, late of H.M. Civil Service. Second Edition. Fcp. 8vo. price 2*s.* cloth.

"A book which brings the so-called mysteries within the comprehension of the simplest capacity."—*Sunday Times.*

"It is clear and concise, and exactly such a text-book as students require."—*Quarterly Journal of Education.*

Text-Book for Architects, Engineers, Surveyors, Land Agents, Country Gentlemen, &c.

A GENERAL TEXT-BOOK for ARCHITECTS, ENGINEERS, SURVEYORS, SOLICITORS, AUCTIONEERS, LAND AGENTS, and STEWARDS, in all their several and varied Professional Occupations; and for the Assistance and Guidance of Country Gentlemen and others engaged in the Transfer, Management, or Improvement of Landed Property; together with Examples of Villas and Country Houses. By EDWARD RYDE, Civil Engineer and Land Surveyor. To which are added several Chapters on Agriculture and Landed Property by Professor DONALDSON, Author of several works on Agriculture. With numerous Engravings, in one thick vol. 8vo, price 1*l.* 8*s.* cloth.

CONTENTS.

To which are added Fourteen Chapters

ON LANDED PROPERTY. By PROFESSOR DONALDSON.

"No Englishman ought to be without this book."

EVERY MAN'S OWN LAWYER; a Hand-Book of the Principles of Law and Equity. By A BARRISTER. 8th Edition, carefully revised, including a Summary of the New Bankruptcy Laws, the Fraudulent Debtors Act, the Reported Cases of the Courts of Law and Equity, &c. With Notes and References to the Authorities. 12mo, price 6*s*. 8*d*. (saved at every consultation), strongly bound.

Comprising the Rights and Wrongs of Individuals, Mercantile and Commercial Law; Criminal Law, Parish Law, County Court Law, Game and Fishery Laws, Poor Men's Lawsuits.

THE LAW OF

BANKRUPTCY.	MORTGAGES.
BILLS OF EXCHANGE.	SETTLEMENTS.
CONTRACTS AND AGREEMENTS.	STOCK EXCHANGE PRACTICE.
COPYRIGHT.	TRADE MARKS AND PATENTS.
DOWER AND DIVORCE.	TRESPASS, NUISANCES, ETC.
ELECTIONS AND REGISTRATION.	TRANSFER OF LAND, ETC.
INSURANCE.	WARRANTY.
LIBEL AND SLANDER.	WILLS AND AGREEMENTS, ETC.

Also Law for

Landlord and Tenant.	Clergymen, Churchwardens.
Master and Servant.	Medical Practitioners, &c.
Workmen and Apprentices.	Bankers.
Heirs, Devisees, and Legatees.	Farmers.
Husband and Wife.	Contractors.
Executors and Trustees.	Stock and Share Brokers
Guardian and Ward.	Sportsmen and Gamekeepers.
Married Women and Infants.	Farriers and Horse-Dealers.
Partners and Agents.	Auctioneers, House-Agents.
Lender and Borrower.	Innkeepers, &c.
Debtor and Creditor.	Pawnbrokers.
Purchaser and Vendor.	Surveyors.
Companies and Associations.	Railways and Carriers.
Friendly Societies.	&c. &c.

"*No Englishman ought to be without this book* . . . any person perfectly uninformed on legal matters, who may require sound information on unknown law points, will, by reference to this book, acquire the necessary information; and thus on many occasions save the expense and loss of time of a visit to a lawyer."—*Engineer*, Jan. 28, 1870.

"It is a complete code of English Law, written in plain language which all can understand . . . should be in the hands of every business man, and all who wish to abolish lawyers' bills."—*Weekly Times*, March 6, 1870.

"With the volume before us in hand, a man may, in nine cases out of ten, decide his own course of action, learn how to proceed for redress of wrongs, or recovery of rights, and save his pocket from the dreaded [illegible] fees and the incalculable bills of costs."—*Civil Service Gazette*, May 23, [illegible].

"We have found it highly satisfactory as a work of authority and reference, and a handy-book of information. There is abundance of cheap and safe law in this work for all who want it."—*Rock*, May 26, 1868.

"A useful and concise epitome of the law, compiled with considerable care."—*Law Magazine*.

"What it professes to be—a complete epitome of the laws of this country, thoroughly intelligible to non-professional readers. The book is a handy one to have in readiness when some knotty point requires ready solution."—*Bell's Life*.